UNCERTAINTY CHEAT SHEET

TYPES OF UNCERTAINTY

- **Systematic:** bias results in one specific direction; due to imperfections in equipment
- **Random:** values obtained will exhibit a spread around a mean value; unavoidable
- **Accuracy:** closeness of a measured value to a standard or known value
- **Precision:** closeness of two or more measurements to each other

EXPRESSIONS OF UNCERTAINTY

Absolute		Relative
$A \pm \sigma_A$	$=$	$A\left(1 \pm \frac{\sigma_A}{A}\right)$

METHODS OF FINDING UNCERTAINTY

- **Granularity** (upper bound): based on coarseness of ruler

$2\sigma =$ distance d between lines of ruler

$$\boxed{\sigma = {}^1\!/_2\, d}$$

- **Digital Granularity:** based on precision of digital instrument

$$\boxed{\sigma = \textit{last digit}}$$

Example: If you read 1.23, then $\sigma = 0.01$

- **Counting:** an estimation based on the Poisson distribution in statistics

$$\boxed{\sigma = \sqrt{N} \text{ for } N \text{ things counted}}$$

- **2/3 Method:** *estimation* from spread of data points
$2\sigma =$ (interval around the mean that contains 2/3 of samples)

$$\boxed{\sigma = {}^1\!/_2 \text{ (spread of 2/3 points)}}$$

- **Standard Deviation:** a rigorous calculations of data set statistics. Like the 2/3 method, it describes the spread of data about the mean

$$\boxed{\sigma = \sqrt{\sum_i \frac{(x_i - \bar{x})^2}{N}}}$$

PROPAGATION OF UNCERTAINTY

- **Multiplication by a Constant**

$$\boxed{\begin{array}{l}\text{Absolute form: } C[A \pm \sigma_A] = (CA) \pm (C\sigma_A) \\ \text{Relative form: } C[A(1 \pm \frac{\sigma_A}{A})] = (CA)(1 \pm \frac{\sigma_A}{A})\end{array}}$$

- **Addition & Subtraction**

$$\boxed{\begin{array}{l}(A \pm \sigma_A) + (B \pm \sigma_B) = (A + B) \pm (\sigma_A + \sigma_B) \\ (A \pm \sigma_A) - (B \pm \sigma_B) = (A - B) \pm (\sigma_A + \sigma_B)\end{array}}$$

- **Multiplication & Division**

$$\boxed{\left[A(1 + \frac{\sigma_A}{A})\right] \times \left[B(1 + \frac{\sigma_B}{B})\right] = (A \times B)\left[1 \pm (\frac{\sigma_A}{A} + \frac{\sigma_B}{B})\right]}$$

$$\boxed{\frac{A(1 \pm \frac{\sigma_A}{A})}{B(1 \pm \frac{\sigma_B}{B})} = \frac{A}{B}\left[1 \pm (\frac{\sigma_A}{A} + \frac{\sigma_B}{B})\right]}$$

- **Functions**

$$\boxed{\sigma_{f(x)} = \frac{f(x + \sigma_x) - f(x - \sigma_x)}{2}}$$

ADDITIONAL NOTES

- Often you are asked to compare measurements / asked if they are within uncertainty of one another. If the are far off, your estimates in uncertainty are probably too small. Where could you improve?
- Treat changing units as multiplication by a constant.
- If you average two numbers with the same uncertainty σ, the uncertainty of the average will also be σ.
- Division with 1 in the numerator: quotient (result) has same RELATIVE uncertainty as the divisor (initial value): $\frac{1}{A \pm \sigma} = \frac{1}{A}(1 \pm \frac{\sigma}{A})$