Gravitational Acceleration on Earth

John Staunton and Aaron Holman
Columbia Physics Department, Columbia University,
538 W. 120th St., New York, NY, 10027, USA.

(Dated: September 9, 2021)

The gravitational acceleration near Earth is a uniform constant, g. It is experienced in every day life, so it is important to have multiple ways to verify its value. We present data from PHET's pendulum module which allows for accurate measurements of the length of the pendulum as well as the period of the pendulum. This results in finding a gravitational acceleration of $g = (10.35 \pm 0.55)$ m/s^2. This measured value is higher than the expected result of $g = 9.8$ m/s^2, and so there is significant error introduced by a human controlled stopwatch.

I. INTRODUCTION

The value of g has been well measured for centuries [1]. There are many famous consequences to uniform gravitational acceleration near the earth, including the equivalence principle [2]. The equivalence principle says that observers in a uniform gravitational field cannot tell that they are accelerating. It is therefore incumbent on us to make sure we have a good value of g and add onto the numerous confirmations of its value, $g = 9.8$.

We will be using a pendulum to find the value for g. The force acting in the angular direction is $mg \sin\theta$, where θ is measured from the vertical. This is equal to mass times acceleration, where acceleration is $l\ddot{\theta}$, where $\ddot{\theta}$ is the angular acceleration. So,

$$ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = mg\sin\theta. \tag{1}$$

For $\theta \ll 10^\circ$, $\sin\theta \approx \theta$. So,

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{g}{l}\theta. \tag{2}$$

This describes oscillatory motion with frequency $\omega^2 = g/l$. Since the period is $T = 2\pi/\omega$, we find,

$$\boxed{l = \frac{g}{4\pi^2}T^2.} \tag{3}$$

In Section II, we discuss the pendulum simulation as well as the two ways in which we will measure g. In Section III, we present the associated data and analysis. In Section IV, we analyze the data and discuss the relevance and implications. Finally, we state our conclusions and outlook at future research in Section V.

II. METHODS

There are two ways to use (3) to find the value g. The first method is to measure the period at various lengths for a pendulum. Then, we can square the values of the period and find a graph of data for l as a function of T^2. This should be a linear function with a slope $g/4\pi^2$. Once we find the slope, we can find the value of g. This is the approach taken in Section III.A.

Another approach would be to simply take multiple measurements at the same length. Then, if one is careful with errors for each of the measurements, we can use standard error propagation techniques to find g with the correct error. This is the approach taken in Section III.B.

In order to make measurements, we use PHET's pendulum lab simulator[1], which affords us the ability to accurately measure the length of the string. We will keep the mass at the end of the weight constant and

[1] https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_en.html

TABLE I.

Length (m)	Length Err (m)	Period (s)	Period Err (s)	Period Sq (s^2)	Period Sq Err (s^2)
0.35	0.01	1.03	0.02	1.06	0.04
0.40	0.01	1.13	0.02	1.28	0.04
0.45	0.01	1.22	0.02	1.49	0.03
0.50	0.01	1.31	0.02	1.72	0.03
0.55	0.01	1.46	0.02	2.13	0.03
0.60	0.01	1.52	0.02	2.31	0.03
0.65	0.01	1.60	0.01	2.56	0.01
0.70	0.01	1.66	0.03	2.76	0.04
0.75	0.01	1.71	0.02	2.92	0.02
0.80	0.01	1.73	0.03	2.99	0.03
0.85	0.01	1.80	0.02	3.24	0.02
0.90	0.01	1.86	0.02	3.46	0.02
0.95	0.01	1.94	0.02	3.76	0.02
1.00	0.01	1.96	0.03	3.84	0.03

Insert caption here.

the angle of release at a constant 5°, which is less than 10°. The mass was also constant at 1 kg. The period was measured using the associated human controlled stop watch on the interactive website.

III. DATA AND ANALYSIS

A. Pendulum at Various Lengths

We measured the period six times for lengths that varied between 0.35 and 1 meter, separated by 0.05 meters. From the six period measurements, we found the error by explicitly finding the standard deviation of the six measured times and utilized it to find the standard error. All errors correspond to two standard errors, or approximately, a 90% confidence interval. Then, we square the period and propagate the error by multiplying it by two, dividing it by the period, and multiplying it by the period squared. The set of all measurements of length, period, and period squared along with their associated errors is presented in Table 1.

In order to obtain the slope, we fit the data with a line, as shown in Figure 1. The best fit line has a slope given by $m \approx 0.262$. The slope of length versus period squared is given in (3) as $g/4\pi^2$, so $g = 4\pi^2 m$. As such, our measured value of g is 10.35. The error in this measurement is given by the covariance matrix, which yields our final result,

$$\boxed{g = (10.35 \pm 0.55)\ \text{m/s}^2.} \tag{4}$$

The goodness of the fit is given by an R^2 value of 0.97. This characterizes a decent fit, however there are clear areas of systematic issues with the plot, namely occurring in the region of small T^2. In this region, the data is systematically large than the fit and it gets worse for shorter periods.

B. Pendulum at a Single Length

In this scenario, using the same fixed mass and angle, but this time fixing the length to be $(0.70 + 0.01)$ m, we measured the period 15 times and computed a standard deviation of 0.08. The error in the period squared is,

$$\sigma_{T^2} = 2\,\langle T\rangle\,\sigma_T = 0.26 \text{ s}. \tag{5}$$

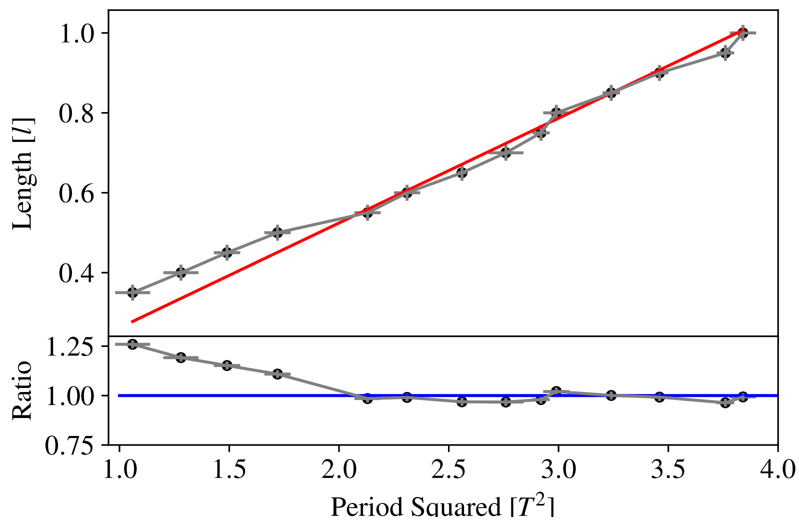

FIG. 1. Best Fit. On the y-axis, we display the values of the length. On the x-axis, we display the values of the period squared. The line is approximately linear and so was fit with a line shown in red, which has a slope $m \approx 0.262$. On the second plot, we show the ratio which describes how good the fit is.

where $T = (1.62 \pm 0.08)$ s, so $\langle T \rangle = 1.62$ and $\sigma_T = 0.08$. As such,

$$T^2 = (2.62 \pm 0.26)\ \text{s}^2. \tag{6}$$

We know from (3) that $g = 4\pi^2 l/T^2$. So, we need to divide l by T^2, which means we divide the means and add the relative errors, yielding

$$\frac{l}{T^2} = (0.27 \pm 0.03)\ \text{m/s}^2. \tag{7}$$

Then, we can multiply $4\pi^2$ to obtain g,

$$\boxed{g = (10.66 \pm 1.18)\ \text{m/s}^2.} \tag{8}$$

This is in agreement, within error bars, with the accepted value of g. Without the large number of measurements, the error bars are certainly larger in this case than they were before. All error propagation estimations were based on the lab manual.

IV. DISCUSSION

In this experiment, we attempted to find the value of acceleration due to gravity near the Earth, as given by g. We found two values for g given by (4) and (8). Both of these values agreed with the accepted value within the error bars, but we note that $10.35 - 0.55 = 9.8$ for the first approach, so it is on lower bound of the 90% confidence interval in this experiment.

The largest source of error was given by the human controlled stop watch. It is difficult to pin down exactly when the pendulum returns to the top of its path, indicating that a sensor might do a better job measuring the period. The location of this sensor would have to manually be placed, which is difficult to do with an interactive experiment.

We additionally note that the assumption was that we were dealing with an ideal pendulum. If we were to deal with a physical pendulum, we would have to account for the mass of the string, or if it was a rigid body, the moment of inertia for the rigid body. Both of these would add complications to our formula for ω, and would therefore alter our measurement for g.

In our second measurement, we note that we used standard deviation to report our errors, but various approximation schemes that are somewhat simpler computationally would yield similar results. Using standard deviation yielded 0.08, but a 2/3rds methods calculations [3] would have yielded 0.06. This would not alter our conclusions about the accepted value of g. In this case as well, the systematic overestimation in periods results from human error in not being able to stop the watch on time. A sensor would have gone a long way in making sure the measurements of the period were accurate.

V. CONCLUSION

The value of g has been measured countless times using various methods. One of the most instructive ways to calculate g is to use a simple pendulum, as it allows for a measurement that solely depends on the period and the length. In doing so, one can find a linear relationship and use the slope to find g. Similarly, one can measure the period a number of times and find a good estimate for g from one value of the length.

In this experiment, we found $g = (10.35 \pm 0.55)$ m/s^2, which in turn found agreement with the accepted value of 9.8 m/s^2, but at the lower end of the confidence interval. We noted that the largest source of error was human error involved in using a stopwatch.

Future experiments should aim to use a sensor to improvement the measurement of g. Measurements on the error can also be increased by measuring the weight of the string used in the pendulum. This will measure just how good our approximation of an ideal pendulum is and can be propagated appropriately into our values of g.

It is also interesting to note that there is an assumption of uniform g. This is a valid approximation when measuring on one location on Earth, but it is interesting to note that this value might differ at different heights on Earth. If this experiment were to improve its precision, it would be interesting to measure how g differs at different heights on the Earth.

[1] C. Hirt, Sten Claessens, Thomas Fecher, et al. *New Ultrahigh-Resolution Picture of Earth's Gravity Field,* (2013), **Geophysical Research Letters**, 40:4279-4283.
[2] S. Carroll, *Spacetime and Geometry*, (2004) **Cambridge University Press**.
[3] F. Pukelsheim, *The Three Sigma Rule*, (1994), **American Statistician** 48:88-91.