Experiment 1-5
Momentum Conservation

Learning Goals

- Understand momentum conservation
- Examine elastic and inelastic collisions
- Study the ballistic pendulum

Introduction

Note: Be sure to note the derivation in the prelab exercises on the ballistic pendulum, it will be necessary for completing the lab.

Momentum is one of the central quantities in physics and, like energy, is bound by a conservation law in the absence of external forces. Although conservation of momentum is less intuitive to many people than conservation of energy, conservation of momentum can be applicable in situations where conservation of energy is not. Sometimes, conservation of energy may not be easily utilized since it can be difficult to keep track of all possible energy contributions (such as energy contribution that deforms the metal of two cars as they collide), but conservation of momentum may still be helpful. For some problems, momentum conservation is essential. We will see that specific problems, like the ballistic pendulum that you will study today, require both energy and momentum conservation for their solution. Others, like collisions on the airtrack, can be solved by momentum conservation where mechanical energy is conserved (elastic) as well as where mechanical energy is not conserved (inelastic). This lab is designed to expose you to several conservation of momentum examples to provide you with an intuitive sense of momentum and some of its important applications[1].

[1] See Chapter 9 of *Fundamentals of Physics* by Halliday, Resnick & Walker.

Theory

Momentum Conservation

Momentum is a vector quantity, so it has magnitude and direction. To add or subtract momenta, one must use the usual rules of vector addition (recall the lab on forces!). In this lab, we deal only with momentum in one dimension, so the vector property is applicable only in the sense that if two objects move in opposite directions, their momenta have opposite signs. This is the source of most mistakes when performing calculations with momenta, so be careful.

Momentum is the product of mass and velocity:

$$\vec{p} = m\vec{v} \tag{1}$$

A fundamental property of nature is that the total momentum components of any closed system[2] are conserved in any physical process *in which external forces are absent.*

The Air Track

Using the gliders on the air track, we will test momentum conservation by measuring momentum of the gliders before and after collisions. Since momentum is the product of mass and velocity, we simply need to measure the mass of the gliders using a balance, and measure the velocities using the apparatus we saw in the velocities lab. We are going to study two different kinds of collisions:

A). **Elastic Collision**. In the first experiment performed on the air track, two gliders with masses m and M will start with velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$. We will use the sonic ranger to measure these velocities following procedures described in section 3. After the collision, the velocities become $\vec{v}_{1f}$ and $\vec{v}_{2f}$. The gliders will undergo nearly perfectly elastic collision, so we can assume both energy and momentum are conserved:

$$m\vec{v}_{1i} + M\vec{v}_{2i} = m\vec{v}_{1f} + M\vec{v}_{2f} \tag{2}$$

$$\frac{1}{2}mv_{1i}^2 + \frac{1}{2}Mv_{2i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}Mv_{2f}^2 \tag{3}$$

After some algebra (done out for you in the appendix), you should be able to solve v_{1f} and v_{2f} from the above two equations, and the result should be

$$\vec{v}_{1f} = \frac{2M\vec{v}_{2i} - (M-m)\vec{v}_{1i}}{M+m}, \quad \vec{v}_{2f} = \frac{2m\vec{v}_{1i} + (M-m)\vec{v}_{2i}}{M+m} \tag{4}$$

Note: you must make sure to use the correct sign in the expressions for v_{1f} and v_{2f} since velocities are vectors. Be careful when interpreting the graphs on DataStudios

[2] A closed system is one in which the sum of forces <u>external</u> to the specified system is zero.

since both Sonic Rangers will read positive velocities even though the carts travel in opposite directions (you will have to interpret one Ranger as measuring positive velocities and the other as measuring negative velocities).

A special case of the above equations is when the two masses are equal, $M = m$, the equations can be simpified into

$$\vec{v}_{1f} = \vec{v}_{2i}, \quad \vec{v}_{2f} = \vec{v}_{1i} \tag{5}$$

or in other words, the two gliders just exchange their velocities.

B). **Inelastic Collision**. In the second experiment the two gliders will stick together after collision. This is called a perfectly inelastic collision meaning that the kinetic energy is no longer conserved. Momentum, however, is still conserved in this case and, because the two gliders stick together after collision, there is only one unknown final velocity. The momentum conservation equation reads

$$m\vec{v}_{1i} + M\vec{v}_{2i} = (m + M)\vec{v}_f \tag{6}$$

where $\vec{v}_f$ is the common final velocity of the two gliders together.

The Ballistic Pendulum

A ballistic pendulum is an instrument usually used to indirectly measure the velocity of a projectile. A fast metal ball strikes a stationary pendulum (in our case a small cup with a latch on the top) and sticks to it. The initial velocity of the ball can be deduced by observing how far the pendulum swings after the collision. In this experiment, we will measure angular displacement of the pendulum, and use that value to estimate the metal ball's initial velocity. We can then compare the measurement with the premeasured velocity data.

To analyze this problem, it is easiest to start by splitting it into two parts. In the first part, consider the metal ball and pendulum in the time right before and right after the collision. The pendulum is initially at rest at its equilibrium point, whereas the metal ball travels with an initial velocity and therefore nonzero kinetic energy. When the ball strikes the pendulum, they stick together and move at the same velocity. But because an unknown part of the initial kinetic energy is lost via vibrations, we cannot directly calculate the pendulum's subsequent motion from conservation of mechanical energy. Instead, we can use conservation of momentum, since during the small time of the collision there are no external forces acting in the direction of motion.

Quantitatively, consider the ball to have a mass m and initial velocity $\vec{v}$, and the pendulum to have a mass M (concentrated completely at the pendulum's end) and initial velocity of 0. The total initial momentum is mv. When the projectile and the pendulum stick together, they have total mass $m + M$ and we define their combined velocity to be $\vec{V}$. The relationship from conservation of momentum is:

$$m\vec{v} = (m + M)\vec{V} \tag{7}$$

So if we know the initial velocity of the projectile $\vec{v}$, we can calculate the combined velocity of the projectile and pendulum $\vec{V}$.

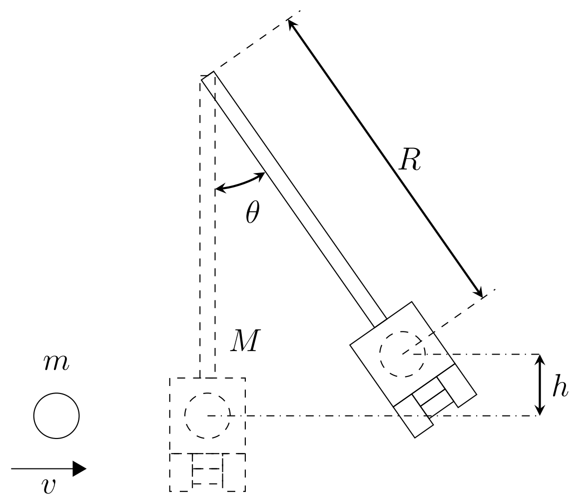

Figure 1: Schematic Diagram for the Ballistic Pendulum

For the second part of the problem, consider the motion of the pendulum after the collision. At this point we *cannot* use conservation of momentum because an external force (gravity) acts on the pendulum more and more in the direction of the pendulum's motion. However, we *can* use energy conservation, because there are no uncontrollable energy losses (like vibrations or friction). Conservation of energy requires the initial kinetic energy of the pendulum system to equal the potential energy at the top of the swing. Which is mathematically given as follows:

$$\frac{1}{2}(m + M)V^2 = (m + M)gh \tag{8}$$

Here, h is the height of the combined (pendulum bob + metal ball) above their lowest point. This relation permits us to calculate the maximum height h that the pendulum reaches from the combined velocity V , which was calculated from the initial velocity of the projectile in the first part.

Due to the design of our pendulum, we have one final step. We will not be measuring the height h which the cup in the pendulum rises to, but rather the angle θ through which it sweeps. By examining figure 1, it is clear that h can be represented as a function of θ in the following way:

$$h = R(\cos\theta_0 - \cos\theta) \tag{9}$$

We now have everything we need to solve for the initial velocity v of the metal ball in terms of the angle θ swept through by the pendulum. Make sure to complete this exercise before lab!

Procedure

The Air Track

Set-up

The air track should be set up according to Figure 2 with a sonic ranger at either side of the air track, both connected to the data-collecting computer. Remember the sonic rangers should be set to "cart" mode for better accuracy by moving the switch at the top to the cart icon. Prior to taking any data, you should use the leveling feet to level the air track so that a glider placed on it would drift as little as possible. Note that a small amount of drifting might be inevitable because there might be some bowing or sagging of the track due to its weight.

Please handle the gliders with care! Don't put them on the track without air flowing and store them only on the felt covered holders provided. Make sure that the bumpers are inserted on both ends of the gliders, and don't make violent collisions! (this also compromises your data).

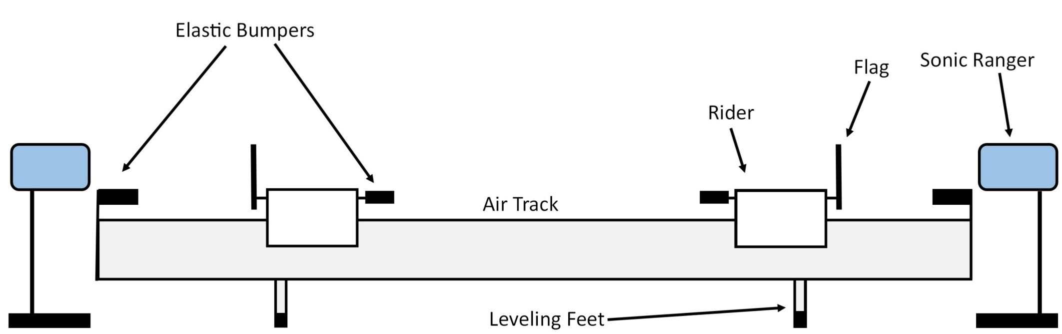

Figure 2: Schematic Diagram of the Air Track Setup

In this experiment we use the computer in the lab to collect data. To log on, use the user name **student** and password **student**. On the desktop screen, click on the MOMENTUMLAB icon to load the software we use for this part of the lab.

To begin taking data, single click on the **Start** button. As the data is being collected, the graph will dynamically display the current velocities of the two gliders. To stop taking data, single click on the **Stop** button. After analyzing the data, you may want to reset the graph by clicking on the menu **Experiment** → **Delete Last Data Run**.

Make sure when exiting DataStudios, **do not** save your current settings.

Elastic Collision of Two Gliders

In the air track experiment, we observe both elastic and inelastic collisions. The air track provides a steady airflow so that the gliders move along a layer of air and the effects of friction are negligible. We use bumpers and elastic rubber bands on the gliders to provide almost perfectly elastic collisions.

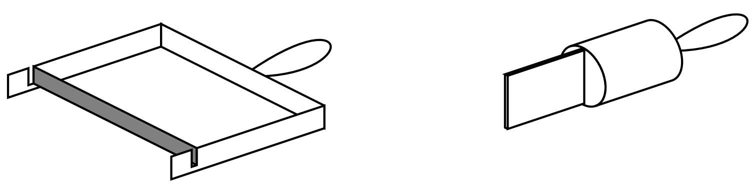

Figure 3: Rubber Band Bumpers and Bumper Plates

NOTE: It is recommended that you **ignore the error on masses**, since they are so much smaller in comparison.

1. Insert an elastic rubber band bumper to the end of one glider (Figure 3), and insert a bumper plate to the end of the other glider.
2. Measure the masses of the gliders with these accessories on, without any additional mass first. Record the masses.
3. Put the two gliders onto the air track and make sure that the bumper plate is facing the rubber band bumper, and the two flags are facing the sonic rangers respectively, as shown in Figure 2.
4. Click **Start** button to start collecting data and send the two gliders moving toward each other with an arbitrary velocity.

5. Record the velocities of the gliders right before collision v_{1i}, v_{2i}, and right after collision v_{1f}, v_{2f}.

6. Calculate the final velocities of the gliders from the v_{1i} and v_{2i} you measured with error (using equation 4).

Now we would like to study some special cases.

7. Keeping one of the gliders stationary in the beginning, send the other glider to collide with this one.

8. Repeat steps $5-6$ in this case.

Now add 2 additional weights to one of the gliders (make sure to put 1 weight on each side so that it is balanced and won't scratch the air track).

9. Put this heavier glider on the air track and keep it stationary.

10. Send the lighter glider moving toward the heavier one with an arbitrary velocity.

11. Record your observations.

We will now work with the lighter glider initially at rest and send the heavier glider to collide with it.

12. Put the lighter glider on the air track and keep it stationary.

13. Send the heavier glider moving toward the lighter one with an arbitrary velocity.

14. Record your observations.

Inelastic Collision of Two Gliders

Replace the rubber band bumper and the bumper plate with a wax tube and a needle from the accessory box. The needle will stick into the wax tube and the two gliders will stick together after collision.

1. First, keep one glider stationary and send the second glider toward the first one with some arbitrary velocity. Record the velocities of the gliders right before collision v_{1i}, v_{2i}, and right after collision v_{1f}, v_{2f}.

2. Calculate the combined velocity of the two gliders after collision from the v_{1i} and v_{2i} you measured from experiment (using equation 6.6). Include error in your calculations.

3. Repeat steps 1-2 with both gliders moving towards each other. Try a few times to find initial velocities such that both gliders come to rest after collision.

4. Record their initial velocities when the final velocity is zero.

5. Compute the ratio of their initial velocities with error.

The Ballistic Pendulum

Setup

Safety remark: Do not have the projectile launcher loaded while you are taking measurements or setting up. Remember to wear the safety goggles when you are actually doing the experiment.

In this experiment, we will measure the velocity of a projectile from a spring-loaded launcher and use it to predict the behavior of a pendulum with which it is colliding. The components are shown in the following diagram:

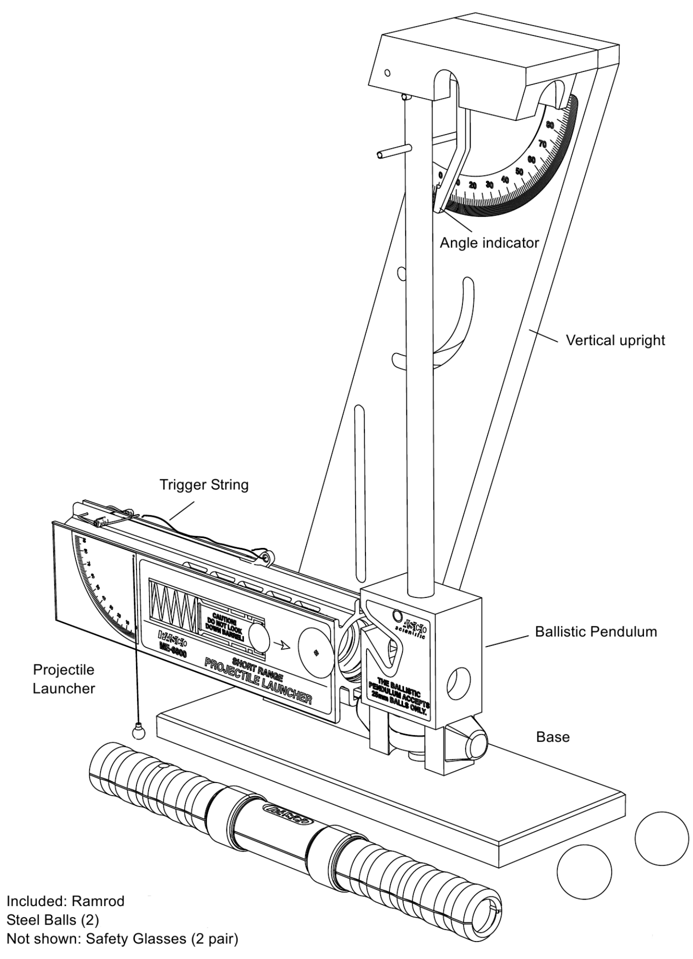

Figure 4: Ballistic Pendulum

- **Launcher** - We will be using a spring-loaded launcher to propel our projectile. There are 3 different velocity settings, i.e. three different positions to which you can compress the spring. We assume that for a single spring position, the projectile will show very little variability in its velocity between launchings (you can convince yourself of this in the next section!).

- **Pendulum** - The pendulum consists of a rigid rod with a cup (and some brass weights) at one end. The cup has a latch which will trap the ball inside upon impact. The rod has a non-negligible mass, but we will assume that our pendulum is a simple pendulum, with all weight concentrated at the end.

Measurements

1. Make sure that that the launcher is attached to the bottom set of slots, as shown in the above diagram. The plumb bob at the end of the launcher indicates the angle of inclination of the launcher.

2. Arrange the launcher so that it lays level, and then load the metal ball into the launcher, compressing the spring to one of the 3 possible load positions.[3]

3. Write down the corresponding quoted velocity of the ball out of the launcher.

4. Once the metal ball is loaded, bring the pendulum down. The black angle indicator should be in position so that it is pushed upward as the pendulum swings forward. It is designed to stick to the position to which the pendulum pushes it, without slipping backward due to gravity.

5. Because the base plate may not rest on a perfectly level surface, the angle indicator at the top of the pendulum may not point to zero even when the pendulum is pointing straight down. Therefore you should first record the initial (small) angle of the pendulum, and then record the final angle of the pendulum, subtracting the two to find the total angle swung through by the pendulum.

6. With the pendulum in position, and the whole setup pointed toward a safe place (away from your fellow physicists!), pull the yellow cord, which releases the catch on the launcher and launches your metal ball.

7. Record the resulting angle of the pendulum, and repeat this procedure for a total of 6 times, recording the final angle θ_{f} each time.

8. Find the average final angle $\theta_{\mathrm{f,ave}}$ with error. Error can be found using the 2/3 method.

[3] Note that during the loading, the pendulum can be clipped to the 90 degrees position so that it stays out of the way.

9. Use the electric scale and your ruler to measure the masses of the ball and the pendulum, as well as the pendulum's length. In order to measure these quantities, you can unscrew the pendulum and take it off from the base.

10. Using your derived expression, calculate the initial ball velocity v_i with error found by propagating uncertainties in θ. To do so, you will need to find the uncertainty in $\cos(\theta)$ which can be found as follows:

$$\sigma_{\cos(\theta)} = \left| \frac{\cos(\theta + \sigma_\theta) - \cos(\theta - \sigma_\theta)}{2} \right| \tag{10}$$

Note, the uncertainty in θ comes from the 2/3 method as discussed above.

11. Repeat steps above with a different compression for the spring, i.e. different initial velocity.

Discussion

Collisions

1. For the cases where you were asked to record and subsequently calculate velocities, do your derived results agree with the final velocities you recorded using the sonic ranger?

2. Qualitatively explain what you would expect to happen when you keep one glider stationary and send the other to collide with it with an arbitrary velocity. How do your expectations differ when the two gliders have equal masses and when they don't? Do your expectations agree with what you observed?

3. For the case of unequal masses, what would you expect to happen if the heavier glider is extremely massive, say with mass 100 times larger than the lighter one? Consider both the case when the heavier glider is moving and when it's stationary.

4. In the very last inelastic collision you performed, you were asked to calculate the ratio of initial velocities. Explain whether your value for the ratio is what you would expect. Compare the experimental velocity ratio to a theoretical prediction from the masses of the gliders and discuss any discrepancies.

The Ballistic Pendulum

1. Does your calculated initial ball velocity agree with what is labelled on the pendulum within error?

2. Discuss the major sources of error in determining the initial ball velocity.
3. Which of your result agrees better with the labelled value?
4. In making our prediction, we assumed a simple pendulum (i.e. one with all mass concentrated at the end). Were you to make a more careful calculation, what steps in our derivation would require modification?

Appendix

It is often difficult to know how to actually do out the algebra to find the velocities using conservation of momentum and energy in an elastic collision, so I will do that here. For ease of reading, I will note both the conservation of momentum and conservation of energy equation below,

$$\begin{aligned} m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} &= m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}, \\ \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 &= \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2. \end{aligned} \tag{11}$$

Let's begin by grouping the masses,

$$\begin{aligned} m_1\left(v_{1i} - v_{1f}\right) &= m_2\left(v_{2f} - v_{2i}\right), \\ m_1\left(v_{1i}^2 - v_{1f}^2\right) &= m_2\left(v_{2f}^2 - v_{2i}^2\right). \end{aligned} \tag{12}$$

Now divide the first equation by the second equation and you will get

$$\frac{v_{1i}^2 - v_{1f}^2}{v_{1i} - v_{1f}} = \frac{v_{2f}^2 - v_{2i}^2}{v_{2f} - v_{2i}}. \tag{13}$$

Note that the numerators can be factored. For example, $v_{1i}^2 - v_{1f}^2 = \left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right)$. One of those factors will cancel out with the denominator. So, we get

$$v_{1i} + v_{1f} = v_{2f} + v_{2i} \tag{14}$$

Now we have two *linear equations*,

$$\begin{aligned} v_{1f} - v_{2f} &= -v_{1i} + v_{2i}, \\ m_1v_{1f} + m_2v_{2f} &= m_1v_{1i} + m_2v_{2i}. \end{aligned} \tag{15}$$

Let's solve for v_{1f} first. To do that I need to eliminate v_{2f} I do that by multiplying the top equation by m_2 on both sides and adding the two equations together. This yields,

$$\left(m_1 + m_2\right)v_{1f} = \left(m_1 - m_2\right)v_{1i} + 2m_2v_{2f}. \tag{16}$$

Similarly, I can multiply the top equation by $-m_1$ and add the two equations together to isolate v_{2f},

$$(m_1 + m_2)\, v_{2f} = 2m_1 v_{1i} + (m_2 - m_1)\, v_{2i}. \tag{17}$$

As a result, we get the desired result,

$$\boxed{\begin{aligned} \mathbf{v}_{1f} &= \frac{m_1 - m_2}{m_1 + m_2}\mathbf{v}_{1i} + \frac{2m_2}{m_1 + m_2}\mathbf{v}_{2i}, \\ \mathbf{v}_{2f} &= \frac{2m_1}{m_1 + m_2}\mathbf{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2}\mathbf{v}_{2i}. \end{aligned}} \tag{18}$$