Experiment 1-3
Forces

Learning Goals

- Examine Newton's three laws of motion
- Examine pulleys and springs
- Understand vector addition

Introduction

Forces are an essential element in the study of physics. The concept of force is familiar: effects of forces such as friction and the gravitational force on the human body are part of everyday experience. Thanks to Isaac Newton who proposed his 3 laws of motion, forces are both physically and mathematically well defined; forces can be measured, combined, and evaluated in exact ways. This laboratory experiment is intended to demonstrate a few of the many examples of commonly encountered forces discussed in the lecture course and how Newton's laws can be used to analyze their behavior.

The objective of this lab is to illustrate the properties of several forces and how we can observe them. First, we deal with the vector nature of forces and how static equilibrium requires that the sum of forces on a single object must be zero. The forces arise from the gravitational attraction to the earth (weight) of objects, which are transmitted to a point through string tension.[1]

Next, we work with the force arising by stretching or contracting a spring. This kind of force has more general applications than springs, in that the "elastic" nature of collisions (like a tennis ball bouncing on the floor) arises from spring-like forces. You

[1]See Chapters 5 and 12 of *Fundamentals of Physics* by Halliday, Resnick & Walker.

will measure the quantitative relationship between the force applied to a spring and the subsequent displacement from equilibrium.[2]

Finally, we deal with a dynamic system where we use computer data acquisition to measure the acceleration caused by gravitational and frictional forces acting on a cart traveling down an inclined plane.

Theory

Vector Representation of Forces

In the first part of the experiment, we measure three vector forces and show that when the system is stationary, the sum of all forces is zero. Vectors have both length *and* direction, and can be represented graphically by an arrow – the length corresponds to the magnitude of the vector, while the direction (from tail to head) corresponds to the direction of the vector. To add two vectors graphically, simply place the tail of one vector at the head of the other. Then start from the same origin and attach the two vectors in the reverse order, we must arrive at the same final point. From the parallelogram formed by this procedure, the arrow along the main diagonal represents the sum of the two vectors (The other diagonal is the difference of the two vectors). See figure 1 for an example of graphical addition of two vectors.

Vectors can also be added algebraically. Vectors are typically described in cartesian coordinates, where the x, y, and z components of the vectors are identified, e.g. $\vec{u} = (u_x, u_y, u_z)$. When adding multiple vectors, you must add all vectors in the same coordinate system (i.e. the x component of one vector must point in the same direction as the x component of the other vector, etc...). For example, if we have two vectors described in cartesian coordinates, ($\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$), the components of the sum, or resultant, vector are $\vec{u} + \vec{v} = (u_x + v_x, u_y + v_y, u_z + v_z)$.

In order to add vectors algebraically, we take advantage of the fact that vectors can be split into components at right angles to each other (in the example above, we split vectors into x, y, and z components). This is an important tool we use in physics repeatedly. You may choose, for the convenience of doing the problem, your x, y, and z axes. There is usually a favorable choice of axes in which the problem can be described most easily. See figure 2 for an example of splitting up the force vectors in figure 1 into components and adding them to find the components of the resultant vector.

[2] See Section 7-7 of Halliday, Resnick & Walker.

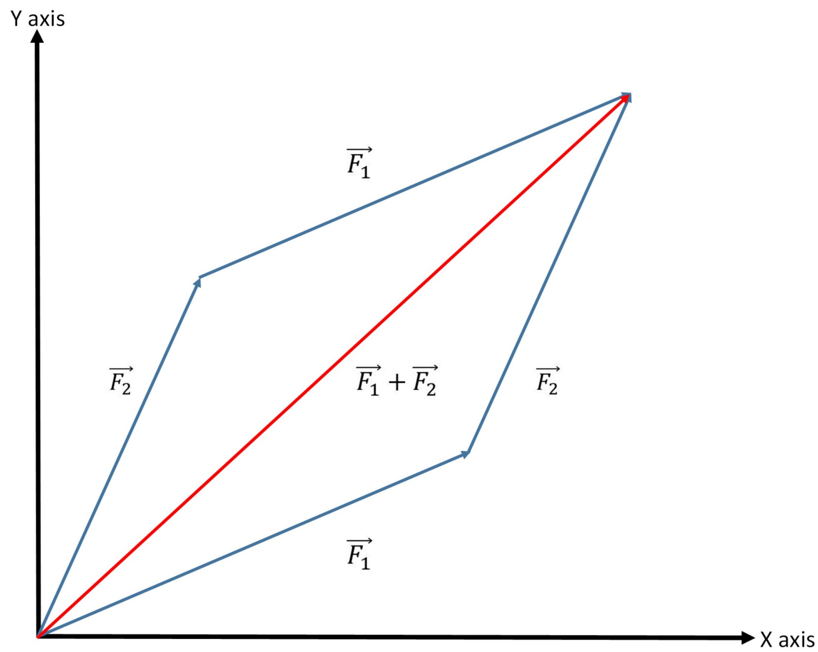

Figure 1: An example of graphical addition of two force vectors F_1 and F_2.

Parallax

In the first part of this lab, we must draw images on paper of strings located some nonzero distance above the paper. It will at first appear that there are many different possibilities of where to draw the images of the strings. If you draw lines on the sheet that seem directly below the strings when your head is in a given position, if you move your head a few centimeters left or right you will see that the lines are no longer covered by the strings. This effect, in which a closer object (here the string) seems to move less relative to a distant background (here the sheet on which you draw the line) is called *parallax*. We will see effects of parallax several times during these labs, so it is important to learn how to handle it.

Where should the lines be drawn? A unique prescription for where to place the lines is provided by the instruction to *always look straight down on the string*. Now we must determine how we know when we are looking straight down on the string. Use a small mirror and place the mirror on the sheet directly under the string. Both the string and the image of the string in the mirror are visible. As your head moves left and right, the mirror image of the string moves relative to the string. When your head is located so that the string and its mirror image exactly overlap, *you are looking straight down*. Make two small marks on each side of the mirror where the image of the string enters and leaves the mirror. Then, remove the mirror and draw a line through

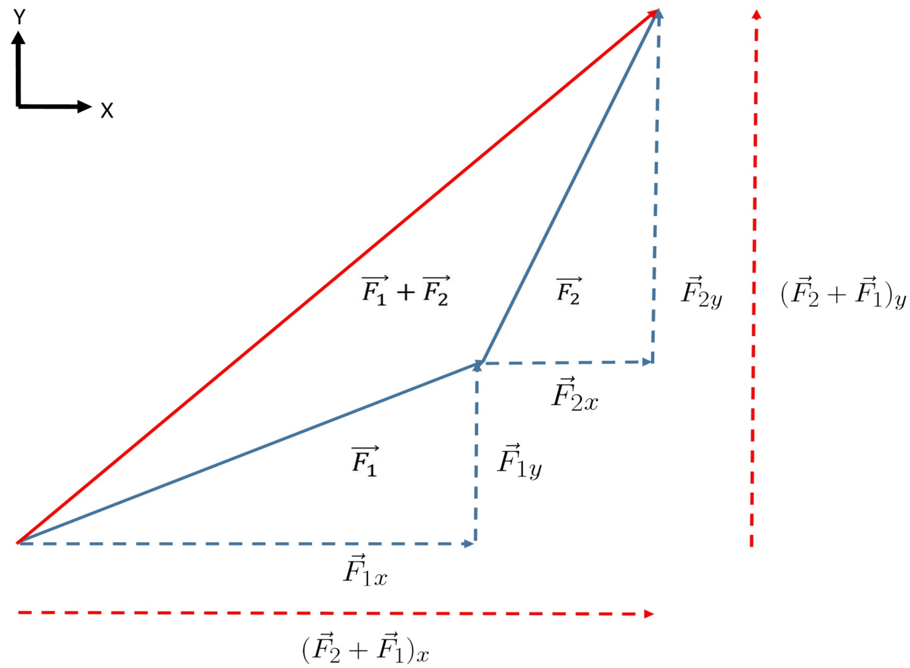

Figure 2: An example of splitting up force vectors into components to add them algebraically.

the two points on the paper with a ruler. This line is the correct image of the string.

Springs

Ideal springs turn out to have a very simple relation between the force F applied to them and the distance s they are stretched from equilibrium. This *linear* relationship, called "Hooke's Law'," is expressed by:

$$\vec{F} = -k\vec{s} \tag{1}$$

where k is known as the "spring constant"[3]. It is a quantity characteristic of each individual spring and its value depends only on the properties of that spring (such as elasticity of the metal, number of coils per length, etc...). In this lab, we will measure the forces necessary to stretch a spring different distances from equilibrium, and from these we will determine the spring constant of the spring.

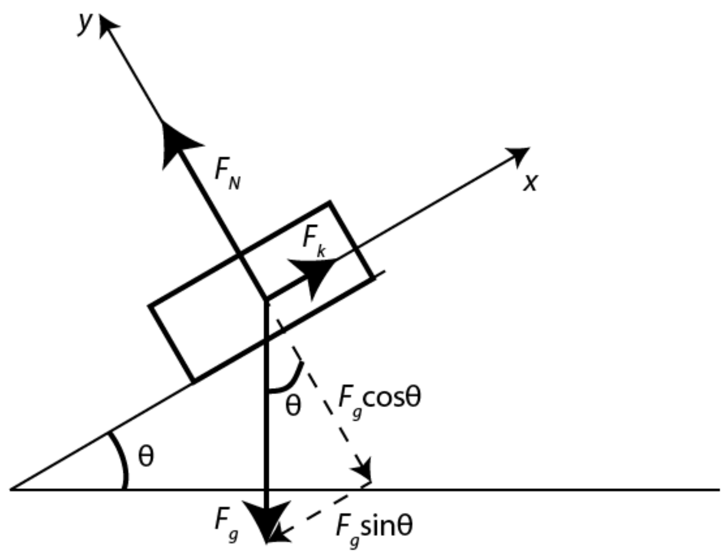

Figure 3: An inclined plane with kinetic friction.

Inclined Plane

In the last part of this lab, we will be looking at the prototypical dynamic system in motion: the inclined plane. When a block slides down an incline there are typically three forces at play: the gravitational attraction to the center of the earth, F_{g}; the normal force that prevents the block from going through the inclined plane, F_{N}; and the kinetic frictional force that slows the descent of the block, F_{k}. Since the net force on the block must be parallel to the surface, using a tilted coordinate system, as in Figure 3, makes it clear that the net force is the following:

$$\vec{F}_{\mathrm{net}} = -F_{\mathrm{g}}\sin(\theta)\hat{x} + F_{\mathrm{k}}\hat{x} = (\mu_k F_N - F_{\mathrm{g}}\sin(\theta))\,\hat{x} \tag{2}$$

Since the normal force, $\vec{F}_{\mathrm{N}} = F_{\mathrm{g}}\cos(\theta)\hat{y}$ and $\vec{F}_{\mathrm{g}} = -mg\hat{y}$ we can derive the following:

$$\vec{F}_{\mathrm{net}} = (mg\sin(\theta) - \mu_{\mathrm{k}} mg\cos(\theta))\,\hat{x} = m\vec{a} \tag{3}$$

Consequently, the acceleration of the block will only depend on the angle of the incline and coefficient of kinetic friction, μ_k. If we solve for μ_k we get the following relationship:

$$\mu_{\mathrm{k}} = \tan(\theta) - \frac{a}{g\cos(\theta)} \tag{4}$$

[3]Note that this denotes the force applied *to* the spring, not the force from the spring due to Newton's 3rd law

Procedure

Addition of Forces

In the first part of the experiment, we deal with three different forces due to hanging masses. The force due to an unknown weight will be exactly balanced by two known forces. We will determine the unknown mass with uncertainty using the graphical addition of vectors and Newton's second law for stationary objects.

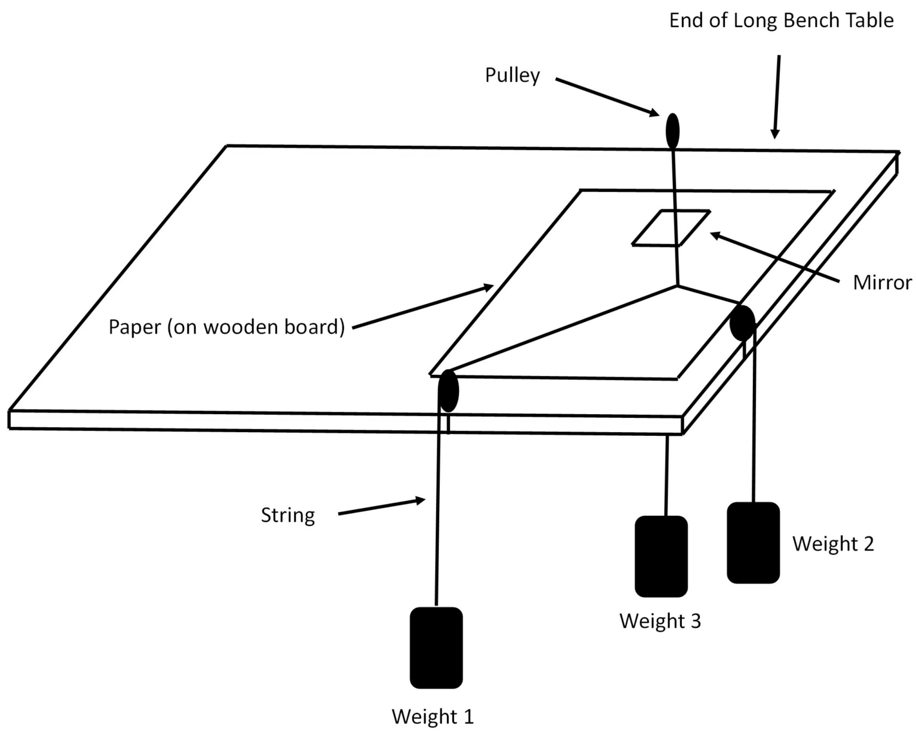

Figure 4: Experimental Setup

The equipment includes a wooden board to which three pulleys are clamped. Over each pulley, we place a string with a weight (making sure the string doesn't slip off the pulley).

1. Choose two different known weights and attach them to two of the strings (you might begin with a 100 g and 150 g weight for example).

2. The three strings are tied together over the board at a "node". Move the node, where the strings join, around gently and try to find the equilibrium position. (The position at which the node no longer moves, or the location to which the node returns if displaced a few cm, represent this equilibrium position. Also

ensure that the weights are not inadvertently hitting the table itself or anything below.)

3. Place a sheet of paper on the wooden board, under the strings, and tape it down to prevent movement while drawing. In order to have room to record the string locations, make sure that the node is approximately in the middle of your paper and the wooden board. (You may need to move the pulleys to accomplish this.)

4. We now need a way to "translate" the weights of the masses to a vector. Draw three line segments on your paper, corresponding to the three strings, using the method of parallax-free reading. These are vector representations of the forces due to each of the weights.

5. After removing your paper, extend the lines so that they intersect at the node.

6. Draw two arrows along the lines corresponding to the strings holding the *known masses*, with lengths proportional to the masses on the strings. Choose a scale that is appropriate for your weights and the size of the paper. A good strategy for making sure the length of the force corresponds to the mass is to define a global quantity "length per unit mass" λ. Then the length of your vector will be $\lambda \cdot m$ where m is the corresponding mass. [4]

7. Add these two vectors graphically using the parallelogram method described above to obtain their resultant force. The resultant of these two vectors should be opposite in direction but equal in length to the vector representing the unknown force (from the unknown mass).

8. Measure the length of the resultant vector. From this length, calculate the magnitude of the unknown mass M with error found by propagating uncertainties in measured lengths l.

9. On the electronic scale, measure the unknown mass M and compare it to the mass determined by the parallelogram method.

Spring

In the second part of the experiment we plot applied force vs. stretch of the spring, make a best-fit straight line and measure its slope. You should understand the physical interpretation of this slope.

[4] For example, we might choose a scale of $\lambda = 1cm/20g$. If we have a mass of $100g$, then the length of the vector must be $5cm$. If done correctly, these final arrows are vectors representing both the magnitude and direction of the forces due to the known weights.

General Comment: Every line is fully determined by its slope and intercept. Whenever you obtain a best-fit line, you should check if these two quantities are reasonable. Also, these two quantities often have a physical interpretation that you should understand.

1. Pick a spring and hang it from a fixed point on the apparatus. You will be using this spring for the entire experiment.
2. Attach a 100 g weight to the bottom of the spring to prevent it from wobbling too much. Adjust the measuring ruler so that a calibration mark lines up with the lower end of the spring.
3. Attach the force meter to the lower end of the spring (below the attached weight) and pull the spring so that it aligns with distances you want to use as measuring points. You can use the mirror from the first part of the experiment to get a parallax-free reading of the position of the spring.
4. Read the force meter to determine the force you are applying to the spring by **pulling it**, and estimate the uncertainty in the reading of the force meter.
5. Record a total of 5 data points (5 distance values and 5 force values) with uncertainty in measured lengths and forces. The uncertainty in measured lengths (forces) can be determined from the precision of the ruler on the force meter (ask yourself if the uncertainty from the upper-bound method is the most appropriate choice for determining uncertainty or if there are other factors at work that add uncertainty).
6. Plot a force vs. distance (stretch) graph in Excel with error bars on both axes.
7. Determine the best-fit line in Excel and determine the slope with error found using the LINEST function.
8. Give the value of the spring constant including error.

Inclined Plane

1. Set up the PAStrack and Inclined Plane Accessory as shown in Figure 5 with a Photogate Head and Photogate Bracket. Elevate the PAStrack to a shallow angle (a few degrees).
2. Check if the inclined plane is leveled. This is done by placing the level across the end of the inclined plane (i.e. perpendicular to the long axis). See if the bubble in the liquid is in between the two lines of the tube. If not, adjust the leveling screws of the inclined plane until the bubble is centered.

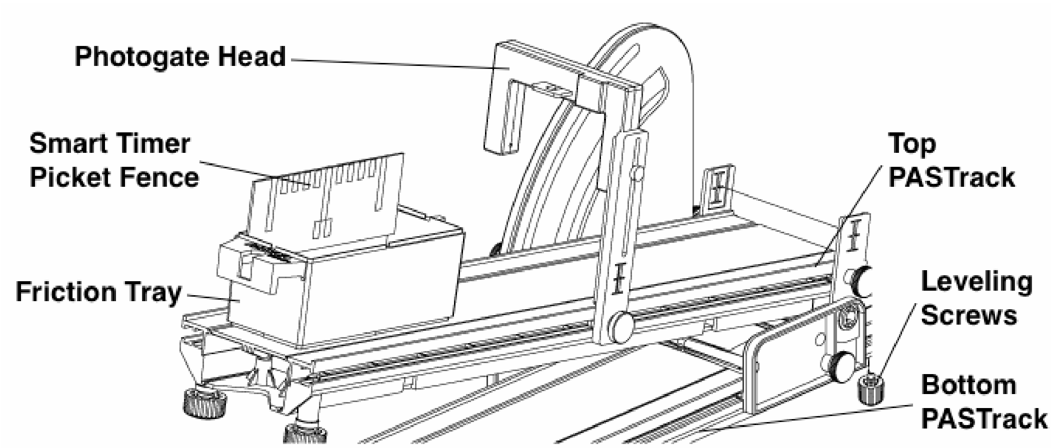

Figure 5: Inclined Plane Apparatus. Angle should be read from top of PASTrack (see physical apparatus).

3. Select the plastic-bottomed Friction Tray and record its bottom material in your data table. Tape the Smart Timer Picket Fence to the Friction Tray so that the 1 cm ‘picket fence’ pattern is at the top (this is the side with *more* ticks).

4. Set the Friction Tray on the PAStrack and adjust the Photogate Head so that the 1 cm ‘picket fence’ pattern of the Smart Timer Picket Fence will interrupt the photogate beam as the Friction Tray moves through the photogate. Put the Friction Tray at the top of the track where it will be released from rest.

5. Raise the angle of the the PAStrack so that when the Friction Tray is released from the top, it accelerates down the plane. We recommend using an incline angle less than 15 degrees. Record the angle in Table 1.

6. Set up the Photogate Head and open the DataStudio file ”Inclined Plane.”

7. Start recording data. Release the Friction Tray from rest. Stop recording data when the tray reaches the bottom of the track.

8. Complete ten trials of the data recording procedure for the first Friction Tray, recording the acceleration after each run (use the statistics tool in Data Studios to find the average acceleration during the run). Make sure to use the same release position in each trial.

9. Calculate the average acceleration for the Friction Tray and use the average acceleration and the angle to determine the coefficient of kinetic friction, μ_k with error due to uncertainty in average acceleration a_{ave}. You can find the uncertainty in average acceleration by determining the standard deviation.

Material	
Angle	
a_{trial1}	
...	
$a_{trial10}$	
a_{ave}	
μ_k	

Table 1: Sample Inclined Plane Data Table.

10. Make sure to record a_{ave} and μ_k in your table.

11. When you are finished taking data, make sure to exit DataStudios and ***DO NOT*** save your settings.

Discussion

Addition of Forces

1. In your sketch of the vector representations of the three weights, are the directions of the resultant vector and the unknown vector exactly opposite? If not, quantify the discrepancy between the two vectors.

2. Are the two masses obtained by measuring the resultant vector and using the mass scale the same? If not, discuss which errors might contribute to the discrepancy.

Springs

1. Discuss the physical meaning of the slope of your graph.

2. What is the value of the intercept of your line (where the distance is zero) with error? How can you interpret the intercept when it is non-zero?

3. Is the value of k you obtained reasonable? Consider the question above in answering this.

Inclined Plane

1. The coefficient of friction for such a plastic-on-plastic combination is $0.2 - 0.4$. How does this quantitatively compare with your result? Does it agree with error?

If not, discuss the largest sources of error in measuring the coefficient of kinetic friction μ_k.