Experiment 1-1
The Simple Pendulum

Learning Goals

- Applying Newton's Second Law
- Understanding period and frequency
- Understanding simple harmonic motion

Theory

In this experiment, we will study the dynamics of a simple pendulum. A simple pendulum is defined as a small mass (known as a pendulum bob), treated as a point mass, suspended from a thin wire considered to be massless. When displaced from equilibrium, the mass will oscillate around the equilibrium point. To analyze the mechanics of the simple pendulum, we use Newton's second law to examine the forces on the pendulum bob (see Figure 1). Looking at Figure 1, we can write down Newton's second law.

$$F = -mg\sin(\theta) \tag{1}$$

$$F = m\frac{d^2}{dt^2}x = ml\frac{d^2}{dt^2}\theta \tag{2}$$

using the small angle approximation $\sin(\theta) \sim \theta$

$$-\frac{g}{l}\theta = \frac{d^2}{dt^2}\theta \tag{3}$$

You may recognize (3) as the equation for simple harmonic motion. In simple harmonic motion, the object (in this case the pendulum) will oscillate about an equilibrium point with a specific frequency f. The frequency can be found from Newton's second law as simple harmonic motion takes the following form.

$$-(2\pi f)^2\theta = \frac{d^2}{dt^2}\theta \tag{4}$$

By comparison of equations (3) and (4), the frequency can be determined in terms of know parameters.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \tag{5}$$

In experiment, we can more easily measure the period of oscillation, which is related to the frequency $\tau = 1/f$. Therefore, we also have an expression for the period of oscillation in terms of know parameters.

$$\tau = 2\pi\sqrt{\frac{l}{g}} \tag{6}$$

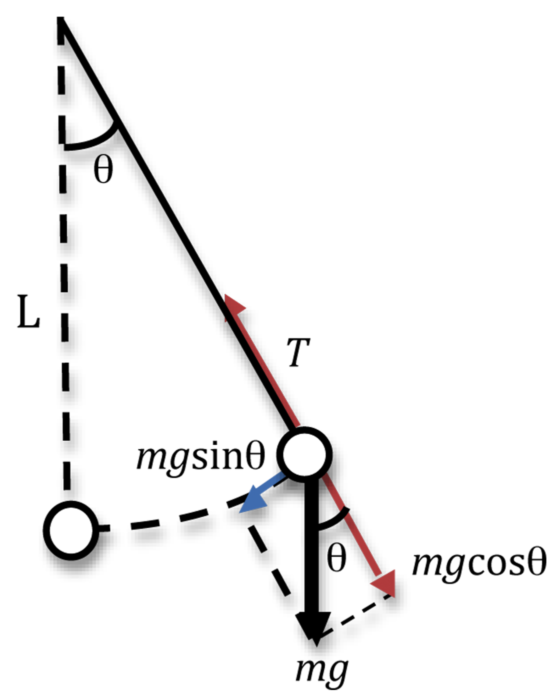

Figure 1: A free body diagram of a simple pendulum undergoing simple harmonic motion

Procedure

You will determine the gravitational constant g through two methods. First, you will measure the time it takes a pendulum to swing through a full cycle, the period of oscillation, and use this to calculate the gravitational constant g. Second, you will measure the period for 5 different pendulum lengths and use the linear fit method to determine the gravitation constant g. Since a number of measurements need to be made, you may wish to perform this experiment in teams of two: one person measures the time values, the other records the results.

- Measurement 1: Calculation of g by measuring multiple periods τ.

  1. Begin by measuring the length of the pendulum with error and recording it in your report.
  2. Let the pendulum swing at some small angle (less than 15 degrees; just release the pendulum, pushing it will skew measurements) and measure the period of the motion. Try to start and stop the stopwatch at the apex of the motion. A full period is the time it takes to travel from one maximum point to back to the same point. Take 18 measurements of the period and record the data in an excel document.
  3. Use excel functions to determine the error in measured period using two methods: first, calculate the standard deviation, then estimate the error using the 2/3 method. Use standard deviation as the error in later calculations.
  4. Use the following formula to determine the gravitational constant g with error found by propagating uncertainties in pendulum length l and period τ.

$$\frac{2 \cdot \pi}{\tau} = \sqrt{\frac{g}{\ell}} \tag{7}$$

- Measurement 2: Calculation of g by measuring the period as a function of pendulum length.

  1. Measure the length of the pendulum with error and record it in your report.
  2. Let the pendulum swing at some small angle (less then 15 degrees) and measure the period of the motion. Try to start and stop the stopwatch at the apex of the motion. Make a single measurement of the period τ.
  3. Repeat steps 1-2 for 5 different pendulum lengths ℓ (use loops provided on string).
  4. In excel, plot the length of the pendulum ℓ versus period squared τ^2 with error bars on both axes. The error bars in ℓ and τ can be determined from the precision in the ruler/stopwatch (make sure to correctly propagate the error in squaring τ).

5. Use excel to fit your plot with a line and use the generated slope to determine the gravitational constant g. Include error in g using the LINEST method you learned last week.

Discussion

1. Do the two methods of determining the error in measured period (2/3 vs. standard deviation) in the first part of the experiment give a similar result? Which is more reliable and why?
2. Do your calculated values of g agree with the expected value ($g = 9.81m/s^2$)?
3. What are the major sources of error in each part of the experiment?
4. How would you improve this experiment?

Applications

You read of a certain test intended to indicate a particular kind of cancer. The test gives you a positive result for $(80 \pm 10)\%$ of all persons tested who really have this kind of cancer (true positives). But the test also gives you a positive result for $(2 \pm 1)\%$ of all healthy persons (false positives). Now you read a publication where the author performed this test on $10,000$ workers that deal with a certain chemical. The author got 400 positive samples from these workers and claims that this is strong evidence that this particular chemical enhances the development of this kind of cancer since it is known from literature that only $(1 \pm 0.5)\%$ of the population are expected to have this kind of cancer. How reliable is the claim of the author?

<u>Numerical Answer</u>:

If one assumes that the 10,000 workers would mirror the average population, then there should be:

$$10000 \times (0.010 \pm 0.005) = 100 \pm 50 \tag{8}$$

persons having this cancer. Of them, the test gives:

$$(0.8 \pm 0.1) \times (100 \pm 50) = 80 \pm 50 \tag{9}$$

positive results (true positives).

There are then 9900 ± 50 persons expected not to have this kind of cancer. Of them:

$$(0.02 \pm 0.01) \times (9900 \pm 50) \approx 200 \pm 100 \tag{10}$$

give positive results (false positives).

The total number of positives in the average population is therefore 280 ± 150.

So how do you judge the author's conclusion of "strong evidences"?

If you wanted to design a new test using the same procedure but to arrive at a stronger conclusion, and you could either increase the rate of true positives or decrease the rate of wrong negatives, which would you choose?

Reference: Paul Cutler: Problem Solving in Clinical Medicine, Chapter 5, Problem 5 (modified).