Experiment 1-6
Torque, Rotational Inertia, and Angular Momentum Conservation

Learning Goals

- Measure rotational motion with distinct inertias
- Observe conservation of angular momentum

Introduction

As you learned in lecture and the second lab, forces are essential in studying the motion of objects through space. However, we find a description of an object’s motion using purely Newton’s three laws of linear motion are only appropriate for describing how the center of mass moves through space. There is no consideration of objects moving while the center of mass is stationary. For example, an object rotating about its center of mass has no linear kinetic energy since the center of mass is stationary, but it does have rotational kinetic energy since the constituent parts of the object are moving with respect to the center of mass. Just as with Newton’s laws of linear motion, we have corresponding laws of rotational motion where force is replaced by its rotational counterpart torque, mass is replaced by moment of inertia (also known as rotational inertia), and linear acceleration is replaced by angular acceleration. The purpose of this experiment is to experimentally verify the rotational inertia of many common objects discussed in lecture including a point mass, a disk, and a ring. We will accomplish this by measuring the rotational motion of the objects under some known torque. We will use computer data acquisition to facilitate detailed analysis of the object’s rotational motion. We will also examine angular momentum conservation by observing the effect of changing the rotational inertia of the system without exerting torque on the system.
Theory

Rotational Inertia

Rotational inertia $I$ is defined via the counterpart of Newton’s Second Law as it applies to rotating bodies,

$$\vec{\tau} = I\vec{\alpha}$$  \hspace{1cm} (1)

where $\vec{\tau}$ is the net torque operating on a rotating body, giving it angular acceleration $\vec{\alpha}$. Thus $I$ is the constant of proportionality between the torque and the angular acceleration. In general, the rotational inertia depends not only on the mass of the rotating body, but also on how that mass is distributed about the axis of rotation. The theoretical rotational inertias of the relevant shapes for this experiment are given in Table 1.

To find the rotational inertia experimentally, a known torque is applied to the object and the resulting angular acceleration is measured. Figure 1 offers a diagram of the Complete Rotational System that will be used in this experiment. It consists of a cast iron base that connects to a rotating platform, where we will attach objects such as a disk or ring in order to measure their rotational inertia. A lever arm with a photogate head can be set up such that a hanging mass (descending under the influence of gravity) exerts a torque on the rotating platform. Figures 3 and 6 also provide insight into the relevant parts of the Complete Rotational System, and how it will be used.

Since $\vec{\tau} = I\vec{\alpha}$,

$$I = \frac{|\tau|}{|\alpha|}$$  \hspace{1cm} (2)

where $\vec{\alpha}$ is the angular acceleration which is equal to $\vec{a}/r$ and $\vec{\tau}$ is the torque caused by the weight hanging from the thread which is wrapped around the step pulley below the rotating platform. From Newtonian mechanics we remember $\vec{\tau} = \vec{r} \times \vec{F}$ (or $|\tau| = |r||F|\sin(\theta) = |r||F|$ if $\vec{r}$ and $\vec{F}$ are perpendicular), so we find

$$|\tau| = |r||T|$$  \hspace{1cm} (3)

where $r$ is the radius of the step pulley about which the thread is wound and $T$ is the tension in the thread when the apparatus is rotating.

Applying Newton’s Second Law for the hanging mass $m$ gives

$$\sum \vec{F} = m\vec{g} - \vec{T} = m\vec{a}$$  \hspace{1cm} (4)
where \(|g| = 9.81 \, \text{m/s}^2 = 981 \, \text{cm/s}^2\) is the gravitational acceleration an objects feels while on the surface of the Earth. In this lab we recommend using grams instead of kilograms, and centimeters instead of meters. This will simplify the algebra slightly.

![Figure 1: A setup used to determine the rotational inertia in this experiment.](image)

Solving for the tension in the thread gives

\[
|T| = m(|g| - |a|) = m(|g| - |\alpha|r).
\]

(5)

If we assume a frictionless system, the net torque would be

\[
\sum \vec{\tau} = m\vec{g}r - m\vec{\alpha}r^2 = I\vec{\alpha}
\]

(6)

\[
I = \frac{m|g|r}{|\alpha|} - m r^2
\]

(7)

Once the angular acceleration of the rotating platform is determined the torque can be obtained for the calculation of the rotational inertia.

**Angular Momentum Conservation**

If you recall from linear Newtonian motion, when no external forces act on a system, the total linear momentum of the system is constant. A similar statement can be said for rotational motion. When no external torques act on a system, the total angular momentum is constant. During the angular momentum portion of the experiment a ring will be dropped onto a rotating disk, thereby changing the system’s rotational inertia. Since there is no net torque on the system, there is no change in angular momentum. Angular momentum is conserved

\[
\vec{L}_i = I_i \vec{\omega}_i = I_f \vec{\omega}_f
\]

(8)
where $I_i$ is the initial rotational inertia and $\omega_i$ is the initial angular speed, while $I_f$ is the final rotational inertia and $\omega_f$ is the final angular speed. The initial rotational inertia is that of a disk, whereas the final rotational inertia is that of a disk and a ring.

\[
I_i = \frac{1}{2}M_1R^2 \tag{9}
\]

where $M_1$ is the mass of the disk and $R$ is its radius. The final rotational inertia $I_f$ is the combination of a disk and a ring

\[
I_f = \frac{1}{2}M_1R^2 + \frac{1}{2}M_2(r_1^2 + r_2^2) \tag{10}
\]

where $M_2$ is the mass of the ring and $r_1$ and $r_2$ are the inner and outer radii of the ring, respectively.

\section*{Procedure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Diagram of the necessary parts to level the apparatus.}
\end{figure}

\section*{Level the apparatus}

The accuracy needed to correctly measure the moments of inertia requires the apparatus to be extremely level. To level the base, perform the following steps:

1. Adjust the position of the 300 g square mass so that its center is at the 22 cm mark by loosening the screw, sliding the mass along the track, and tightening the
screw so the mass will not slide. Rotate the track until the 300g mass is above the left foot of the “A” base. See the left side of Figure 2.

2. Adjust the leveling screw on the right side of the “A” base until the end of the track with the square mass is aligned over the leveling screw on the left other leg of the base. See the left side of Figure 2.

3. Rotate the track 90 degrees so it is parallel to the right edge of the “A” base and adjust the left leveling screw until the track will stay in that position (parallel to the right edge of the “A” base). See the right side of Figure 2.

4. The track should now be level and should remain at rest in any orientation.

Rotational inertia of a point mass

1. Remove the 300 g square mass, attach a thread to the middle spindle of the step pulley and hang the thread over the 10-spoke pulley. Allow the string to reach the floor. Use a caliper to measure the radius of the middle spindle, $r_{\text{spindle}}$.

2. Open the ”Torque, Rotational Inertia, and Angular Momentum” DataStudio program.

3. Attach a mass to the thread (we suggest 50g; note that the mass-holding apparatus weighs approximately 5g) and wind the middle spindle until the mass is hanging right below the pulley.

4. Allow the rotating platform to rotate freely and start acquiring data by clicking the “Start” button in DataStudio as shown in Figure 4. Stop data acquisition before the mass hits the floor by clicking the same button.

Figure 3: The setup for above section.
5. Highlight a linear portion of the angular velocity graph as shown in Figure 5 and perform a linear fit by clicking the “Fit” drop down menu and selecting “Linear Fit” as shown in Figure 4.

6. Record the slope of the angular velocity graph, which is the average angular acceleration $\alpha_{\text{avg}}$, in Table 2 in the “No Mass on Track” row and compute the experimental rotational inertia with error.

7. Calculate the theoretical rotational inertia of the system $I_{\text{theory}}$ with error found by propagating uncertainties in the length of the rod, $L$. Record your result in Table 2 in “No Mass on Track” row.

**Note for Steps 8 and 9:** Moments of inertia add together, so to calculate the rotational inertia of only the point mass, we must subtract the “No Mass on the Track” rotational inertia from the “300g Mass at 22cm” and the “300g Mass at 11cm” rotational inertias.

8. Attach the 300 g mass back onto the track and slide it along until it is centered at the 22 cm mark. Repeat steps 3 through 7 using the same suspended mass (we suggest 50g) and fill in Table 2 in the “300g Mass at 22 cm” row while
propagating uncertainties from the radius of the point mass, $R_{\text{inertia}}$, and the length of the rod, $L$.

9. Loosen the 300 g mass and slide it along the track until it is centered at the 11 cm mark. Repeat steps 3 through 7 using the same suspended mass (we suggest 50 g) and fill in table 2 in the “300g Mass at 11 cm” row while propagating uncertainties from the radius of the point mass, $R_{\text{inertia}}$, and the length of the rod, $L$.

Rotational inertia of a disk

1. Remove the straight track from the “A” base by loosening the screw under the track and lifting it from the center shaft. Put the straight track aside. Position the rotational disk horizontally on the center shaft as shown in diagram of the upper-rightmost corner of Table 1. The side of the disk that has the indentation for the ring should be up and align the ”D” shaped hole of disk with ”D” shape of shaft.

2. Attach a hanging mass (we suggest 50 g) to the thread and wind the middle spindle until the mass is hanging right below the pulley.
3. Allow the rotating platform to rotate freely and start acquiring data. Stop data acquisition before the mass hits the floor.

4. Highlight a linear portion of the angular velocity graph and perform a linear fit.

5. Record the slope of the angular velocity graph (the average angular acceleration $\alpha_{\text{avg}}$), in Table 2 and compute the experimental rotational inertia with error.

6. Calculate the theoretical rotational inertia of the system $I_{\text{theory}}$ with error found by propagating uncertainties in radius of disk, $R_{\text{disk}}$, or length of rod, $L$. Record your result in table 2 in the “Flat Rotational Disk” row.

7. Remove the disk from the center shaft and rotate it up on its side. Mount the disk vertically by inserting the shaft in one of the two circular holes on the edge of the disk, aligning the flat side of the shaft with the screw. See Figure 6.

8. Repeat steps 2 through 6 using the same suspended mass (we suggest 50g) and fill in Table 2 in the “Rotational Disk Side” row.

![Figure 6: The setup for the above section steps 7 and 8.](image)

**Discussion**

1. Do the experimentally measured rotational inertia $I_{\text{exp}}$ agree with your theoretical calculations $I_{\text{theory}}$ within error?

2. Discuss which sources of error may contribute to incorrect calculations of the rotational inertia.
**Torque, Rotational Inertia, and Angular Momentum Conservation**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Orientation of the Axis</th>
<th>Rotational Inertia</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point Mass</td>
<td>$R$ from point mass</td>
<td>$I = MR^2$</td>
<td></td>
</tr>
<tr>
<td>Uniform disk with radius $R$</td>
<td>Through center of disk, perpendicular to the plane of the disk</td>
<td>$I = \frac{1}{2}MR^2$</td>
<td></td>
</tr>
<tr>
<td>Uniform disk with radius $R$</td>
<td>Along diameter of disk</td>
<td>$I = \frac{1}{4}MR^2$</td>
<td></td>
</tr>
<tr>
<td>Uniform ring with inner radius $R_1$ and outer radius $R_2$</td>
<td>Through center of ring, perpendicular to plane of the ring</td>
<td>$I = \frac{1}{2}M(R_1^2 + R_2^2)$</td>
<td></td>
</tr>
<tr>
<td>Uniform rod with length $L$</td>
<td>Through the center of the rod</td>
<td>$I = \frac{1}{12}ML^2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The rotational inertia along with diagrams for various common objects.

<table>
<thead>
<tr>
<th>Object</th>
<th>$\alpha_{avg}$ (rad/s $^2$)</th>
<th>$I_{exp}$(g · cm$^2$)</th>
<th>Suspended Mass (g)</th>
<th>$I_{theory}$(g · cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Mass on Track</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300g Mass at 22cm</td>
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<tr>
<td>300g Mass at 11cm</td>
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<tr>
<td>Flat Rotational Disk</td>
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<tr>
<td>Rotational Disk Side</td>
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Table 2: Data table for various parameters measured in this experiment.
Conservation of Angular Momentum

1. Position the rotational disk directly on the center shaft as shown in Figure 9. The side of the disk that has the indentation for the ring should be up.

2. Spin the disk with your hand.

3. Start recording data using the "Torque, Rotational Inertia, and Angular Momentum" DataStudio program. After approximately 25 data points have been taken, carefully drop the mass ring onto the spinning disk so that it rests on the indentation. Stop recording data a few seconds after the mass ring is dropped on the disk.

4. Use the velocity graph to determine the angular speed right before the collision ($\omega_i$) and right after the collision ($\omega_f$).

5. Repeat for 3 trials and enter data into Table 3.

6. Calculate the initial and final moments of inertia $I$, using the appropriate expressions for $I$ for the disk and the ring. You may need to measure their masses and
radii. Include error in calculated moments of inertia $I$ by propagating uncertainty in measured radius $r$.

7. Calculate initial and final angular momenta $L$, by multiplying moments of inertia $I$ by the relevant initial and final angular velocities. Include error in angular momentum $L$ by propagating uncertainties in moment of inertia $I$. Enter these values with error into Table 3.

8. Calculate the ratio of the initial angular momentum to the final angular momentum ($R = |L_i|/|L_f|$) for all three trials. Calculate the uncertainty in $R$ by propagating uncertainties in angular momentum.

![Figure 9: The conservation of angular momentum setup mentioned above.](image)

<table>
<thead>
<tr>
<th>$\omega_i$ (rad/s)</th>
<th>$\omega_f$ (rad/s)</th>
<th>$L_i$ (kg.m$^2$/s)</th>
<th>$L_f$ (kg.m$^2$/s)</th>
<th>$R$</th>
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Table 3: Angular velocities and angular momenta before and after collision.
Discussion

Angular Momentum

1. Is angular momentum conserved within error? What value of $R$ should we get if angular momentum is conserved?

2. Why might angular momentum not be conserved (think about the assumption of angular momentum conservation)?

3. What are the main sources of error? Which source of error do you think dominates in this experiment?