I will discuss general constraints on the spectrum of black holes in quantum gravity, coming from invariance under large diffeomorphisms, together with holography. These constraints relate two seemingly different problems: the numerical conformal bootstrap, and the classic sphere packing problem in mathematics. It turns out that constraining the black hole spectrum is exactly equivalent to the sphere packing problem in 8 and 24 dimensions, where it was recently solved by Viazovska in the dramatic culmination of a centuries-long effort by mathematicians.