

Uncertainty Cheat Sheet

EXPRESSIONS OF UNCERTAINTY

Absolute	=	Relative
$A \pm \sigma_A$		$A(1 \pm \frac{\sigma_A}{A})$

TYPES OF UNCERTAINTY

- **Systematic:** bias results in one specific direction; due to imperfections in equipment
- **Random:** values obtained will exhibit a spread around a mean value; unavoidable
- **Accuracy:** closeness of a measured value to a standard or known value
- **Precision:** closeness of two or more measurements to each other

METHODS OF FINDING UNCERTAINTY

- **Granularity** (upper bound): based on coarseness of ruler

$2\sigma =$ distance d between lines of ruler

$$\sigma = \frac{1}{2} d$$

- **Digital Granularity:** based on precision of digital instrument

$$\sigma = \text{last digit}$$

Example: If you read 1.23, then $\sigma = .01$

- **Counting:** an estimation based on poisson distribution in statistics

$$\sigma = \sqrt{N} \text{ for } N \text{ things counted}$$

- **2/3 Method:** *estimation* from spread of data points

$2\sigma =$ (interval around the mean that contains 2/3 of samples)

$$\sigma = \frac{1}{2} (\text{spread of } 2/3 \text{ points})$$

- **Standard Deviation:** a rigorous calculation of data set statistics. Like the 2/3 method, it describes the spread of data about the mean

$$\sigma = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{N}}$$

PROPAGATION OF UNCERTAINTY

- **Multiplication by a Constant**

Absolute form: $C[A \pm \sigma_A] = (CA) \pm (C\sigma_A)$
Relative form: $C[A(1 \pm \frac{\sigma_A}{A})] = (CA)(1 \pm \frac{\sigma_A}{A})$

- **Addition & Subtraction**

$(A \pm \sigma_A) + (B \pm \sigma_B) = (A + B) \pm (\sigma_A + \sigma_B)$
$(A \pm \sigma_A) - (B \pm \sigma_B) = (A - B) \pm (\sigma_A + \sigma_B)$

- **Multiplication & Division**

$$\left[A(1 \pm \frac{\sigma_A}{A}) \right] \times \left[B(1 \pm \frac{\sigma_B}{B}) \right] = (A \times B) \pm (\frac{\sigma_A}{A} + \frac{\sigma_B}{B})$$

$$\frac{A(1 \pm \frac{\sigma_A}{A})}{B(1 \pm \frac{\sigma_B}{B})} = \frac{A}{B} \pm (\frac{\sigma_A}{A} + \frac{\sigma_B}{B})$$

- **Functions**

$$\sigma_{f(x)} = \frac{f(x + \sigma_x) - f(x - \sigma_x)}{2}$$

ADDITIONAL NOTES

- Often you are asked to compare measurements / asked if they are within uncertainty of one another. If they are far off, your estimates in uncertainty are probably too small. Where could you improve?
- Treat changing units as multiplication by a constant
- If you average two numbers with the same uncertainty σ , the uncertainty of the average will also be σ
- Division with 1 in the numerator: quotient (result) has same RELATIVE uncertainty as the divisor (initial value): $1/(A \pm \sigma) = (1/A)(1 \pm \sigma/A)$