Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 16, 2015
1:00PM to 3:00PM
General Physics (Part I)
Section 5.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 5 (General Physics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

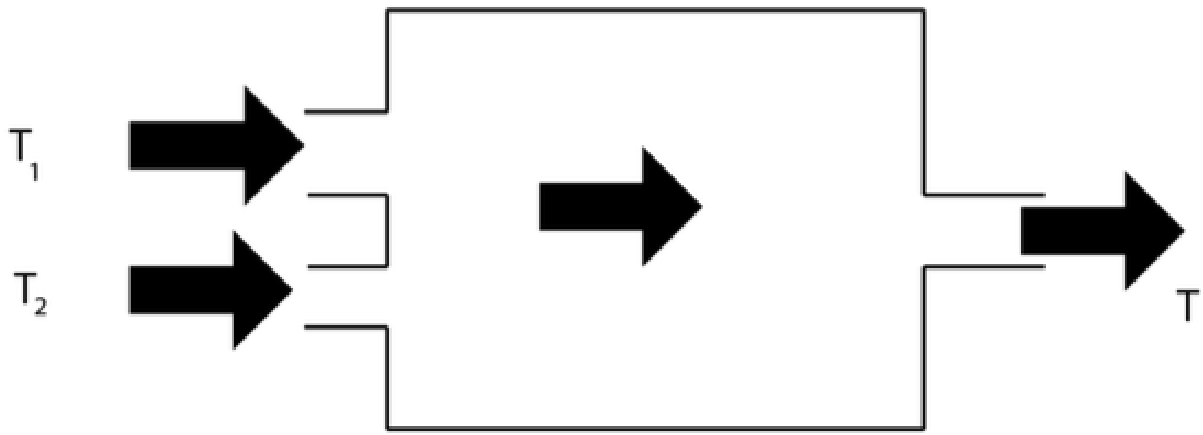
You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

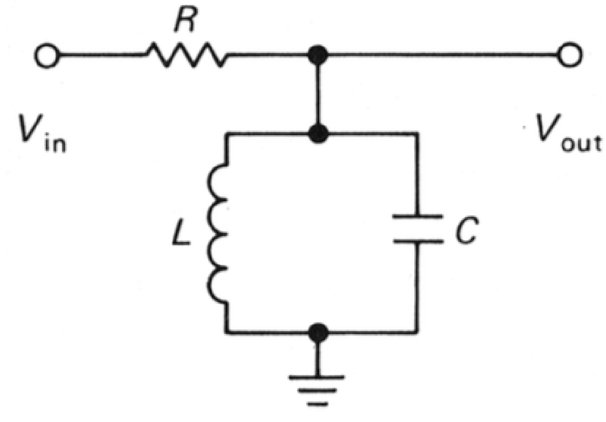
1. A machine only inputs two equal steady streams of hot and cold water at temperature T_1 and T_2. Its only output is a single high-speed jet of water. The heat capacity per unit mass of water, C, may be assumed to be independent of temperature. The machine is in a steady state and the kinetic energy of the incoming streams is negligible.



(a) What is the speed of the jet in terms of T_1, T_2 and T, where T is the temperature of water in the outgoing jet?

(b) What is the maximum possible speed of the jet?

2. Derive an expression for $\left|\frac{V_{out}}{V_{in}}\right|$ as a function of frequency for the resonator circuit below and determine the resonant frequency from this expression.



LC resonant circuit

3. As we go up in altitude h above the ground, the atmospheric pressure $P(h)$ decreases. Assume a simple model in which the atmosphere is isothermal, with ground temperature T_0 and ground pressure P_0. Simplify further by assuming the atmosphere consists of diatomic molecules of two point masses M separated by small, rigid, massless bars. Assume temperatures sufficiently high that these molecules can be treated by classical physics.

   (a) What is the atmospheric pressure as a function of altitude, $P(h)$?

   (b) What is the specific heat per mole of this gas?

   In reality the atmospheric temperature depends on local pressure, $T = T(P)$. Assume the dependence of temperature on pressure is the same as that for a mass of air moving upwards or downwards with non exchange of heat with its surroundings.

   (c) Calculate $T(P)$ with the above assumptions.

   (d) Use your relation $P(h)$ from part a) to determine $T(h)$.

4. The sun is at a distance $D \sim 1.5 \times 10^{13}\,\mathrm{cm}$ from the earth and the gravitational constant $G \sim 7 \times 10^{-8}\,\mathrm{cm^3 g^{-1} s^{-2}}$. In addition, two observations will be useful in what follows. The angular width of the sun as seen from the earth is $\sim 10^{-2}\,\mathrm{rad}$; the intensity of sunlight on a piece of paper at noon is comparable to that from a 100 W light bulb shining on the piece of paper 10 cm away at midnight. From these observations, or any others you care to utilize, make order of magnitude estimates of the following quantities.

   (a) The mass of the sun $M_\odot$

   (b) The solar luminosity $L_\odot$

   (c) The solar radius $R_\odot$

   (d) The surface temperature of the sun, $T_\odot$

   (e) The total kinetic energy of all the particles in the sun (ignoring photons), K

   Suppose there was an explosion deep in the solar interior which added an additional energy ΔE there.

   (f) After a new steady state is established, what is the fractional change in radius of the sun (magnitude and sign) in terms of ΔE and the initial kinetic energy?

   (g) The sun maintains its steady state by nuclear reactions in the stellar core. Suppose these reactions were to cease. From the current solar values of some of the quantities you determined above, estimate the characteristic timescale on which the kinetic energy would change i.e. $K/(\mathrm{d}K/\mathrm{d}t)$.

5. A graduate student constructed a field effect transistor (FET). The operation of this FET device requires that the density of electrons with spin $S = 1/2$ is varied by an external voltage. The student claims that the quantum states of the electrons have energy states that can be represented as:

$$E(j, p_x, p_y) = (j + 1/2)E_z + (1/2m)(p_x^2 + p_y^2)$$

$(j + 1/2)E_z$ represents the energy for motion along the z-direction that is normal to the (x,y) plane. p_x and p_y are the components of electron momentum in the (x,y) plane. $E_z = 0.005\,\text{eV}$. The allowed values of the quantum number j are $j = 0, 1, 2 \ldots$.

When populating states $E(j, p_x, p_y)$, the electrons form a two dimensional electron gas with vanishing impact of Coulomb interactions. The allowed values of electron momentum are subject to quantization within a square of area $A = L^2$, where L is the length of the sides of the square.

(a) The density of states of this non-interacting 2D electron system determines the number of states per unit area in the energy interval dE. Evaluate the density of states.

(b) Assume that the areal electron density is $n = 10^{11}\,\text{cm}^{-2}$. Find the difference between the energies of the lowest- and highest-occupied states at the temperature $T_1 = 0\,\text{K}$. Describe qualitatively the changes that occur when the temperature is raised to $T_2 = 80\,\text{K}$.

(c) Repeat (b) with $n = 10^{13}\,\text{cm}^{-2}$ and $T = T_1 = 0K$.

6. A charged ($q = 10^{-8}$ C) ball is suspended above a large perfectly calm body of salt-water. The diameter of the ball is the same as the distance of the bottom of the ball from the surface of the water (both 1 cm). What will happen to the water under the ball? Provide both a qualitative description and quantitative estimate of the effect.

Solution

(a) The heat in per unit mass is:

$$\Delta Q = \frac{1}{2} c (T_1 - T) - \frac{1}{2} c (T - T_2)$$

For steady state,

$$\frac{v^2}{2} = \Delta Q$$

$$\Rightarrow \quad v = \sqrt{c(T_1 + T_2 - 2T)}$$

(b) Entropy increase is positive

$$\Delta S = \frac{1}{2} c \ln \frac{T}{T_1} + \frac{1}{2} c \ln \frac{T}{T_2} \geq 0$$

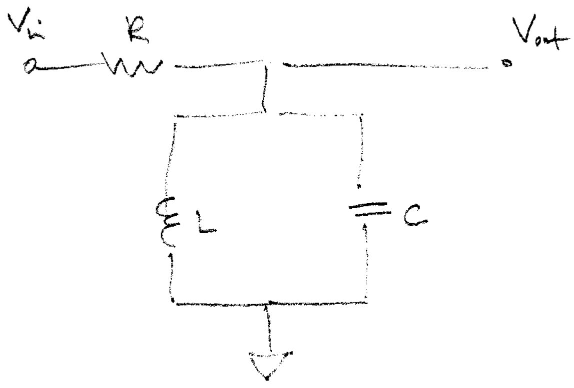
$$= \frac{1}{2} c \ln \frac{T^2}{T_1 T_2} \geq 0$$

$$T^2 \geq T_1 T_2 \quad \text{or} \quad T \geq \sqrt{T_1 T_2}$$

$$v_{MAX} = \sqrt{c(T_1 + T_2 - 2\sqrt{T_1 T_2}}$$

Solution to Problem #2

Johnson
Sec 5 - 2
Solution



$$Z_L = j\omega L$$

$$Z_C = 1/j\omega C$$

$$Z_{total} = R + \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C}} = R + \frac{Z_L Z_C}{Z_L + Z_C}$$

$$Z_{total} = R + \frac{j\omega L / j\omega C}{j\omega L + 1/j\omega C} = R + \underbrace{\frac{j\omega L}{1 - \omega^2 LC}}_{Z_{res}}$$

$$I_{tot} = \frac{V_{in}}{Z_{total}}, \quad V_{out} = I_{tot} Z_{res}$$

$$Z_{total} = \frac{R(1-\omega^2 LC) + j\omega L}{1 - \omega^2 LC}$$

$$V_{out} = V_{in} \frac{Z_{res}}{Z_{total}}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_{res}}{Z_{total}} = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L} \rightarrow \frac{V_{out}}{V_{in}} = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L} \cdot \frac{R(1-\omega^2 LC) - j\omega L}{R(1-\omega^2 LC) - j\omega L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left[R^2(1-\omega^2 LC)^2 + \omega^2 L^2\right]^{-1} \left|\omega^2 L^2 + j\omega L R(1-\omega^2 LC)\right|$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\sqrt{\omega^4 L^4 + \omega^2 L^2 R^2 (1-\omega^2 LC)^2}}{\omega^2 L^2 + R^2(1-\omega^2 LC)^2} = \frac{\omega L}{\sqrt{\omega^2 L^2 + R^2(1-\omega^2 LC)^2}}$$

$$\boxed{\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + R^2 \frac{(1-\omega^2 LC)^2}{\omega^2 L^2}}}}$$

maximum when $(1 - \omega^2 LC) = 0$, $\omega^2 = \frac{1}{LC}$

$$\boxed{f_{res} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$$

MAI - solar problem

Answers to Problem (1)

~~within factor~~ of several of

a 1) $\frac{GM}{d^2} = \left(\frac{2\pi}{1\,yr}\right) d$ $M_\odot \sim 2 \cdot 10^{33}$ g

b 2) $\frac{L_\odot}{4\pi d^2} \sim \frac{10^2 \text{ watts}}{4\pi (10\text{ cm})^2}$ $L_\odot \sim 4 \cdot 10^{33}$ ergs/s

c 3) $\frac{2 R_\odot}{d} \sim 10^{-2}$ $R_\odot \sim 7 \cdot 10^{10}$ cm

d 4) "white hot" $T_s \sim 6 \cdot 10^3$ °K

or

$L_\odot \sim 4\pi R_\odot^2 \sigma_{SB} T_{s}^4$

$\sigma_{SB} \sim 10^{-4}$

e 5) $E_{total} = K + V_G < 0$

virial $K = -\frac{V_G}{2}$

$\therefore E_{T0} = -K$

$\therefore K \sim \frac{G\hat{M}^2}{R_\odot \cdot 2} \sim 2 \cdot 10^{48}$ ergs

1) 1
2) 1
3) 1
4) 1
5) 2
6) 2
7) 2
8) 2
9) 2

e) $\sim \frac{7\times10^{-8}\cdot 4\times10^{66}}{2.7\times10^{16}} \simeq 2\times10^{48}$ erg

f) $\Delta K + \Delta U = E$ ← added energy

$U_f = -2K_f$; $U_i = -2K_i$

$\Delta U = -2\Delta K \Rightarrow \frac{\Delta U}{2} = E$; $-\Delta K = E$

$U \sim -\frac{GM}{R}$ $\quad \frac{GM\Delta R}{2R^2} = E \quad \frac{GM}{2R} = K \Rightarrow \underline{\frac{\Delta R}{R} = \frac{E}{K}}$

g) $\tau^{-1} = \frac{dK}{dt}\Big/K \approx \frac{\Delta K}{K\,t} \qquad \Delta K = -E$

$\tau = \frac{K}{\Delta K/t} \sim K/L_\odot \sim 2\times10^{7}$ yrs

using e) and $L_\odot \sim 2\times10^{33}\frac{\text{erg}}{\text{s}}$

General M. Ruderman

As we go up in altitude (h) above the ground atmospheric pressure ~~dec~~ (P) decreases. In the lower atmosphere ~~so does~~ ~~does the temperature (T) an~~ the atmospheric composition (80 % N_2, 20 % O_2) remains relatively unchanged

Points out of 15

(4)

a) In an overly simple model the atmospheric temperature $T = T_0$, the ground temperature, at all h and the same 2M = the molecular mass for both N_2 and for O_2

In this model what is P(h), the atmospheric pressure as a function of altitude?

~~In P, as a function of P(0) and T_0?~~

4 b) However, we know that T drops rapidly with increasing h (cold mountain peaks; high flying airplanes, etc.). The lower around altitude atmosphere temperatures depend upon ~~the~~ its pressure

just as it would in an isolated large mass of air moving upward or downward

(10) What is T(P) in such a model?

(1) What is T(h) if the P in this model is that of part a)

[Approximate an N_2 (O_2) molecule as two point masses, each of mass m_N (m_o), seperated by a small massless rigid bar. Assume temperatures are sufficiently high that the motions of and in these molecules can be well approximated by classical physics. Neglect all heat flow out of or into the rising (or falling) air mass.]

a) $P = P(h=0)\, e^{-\frac{2Mgh}{k_B T_0}}$

b)

(see next page)

Answers To Problem (2
continued (2)

b) Isentropic flow upward (or downward)

$$\frac{C_v\,dT}{T} + \frac{P\,dV}{T} = 0$$

$+$ $PV = RT$ (C_v is per mol.)

$$\Rightarrow \quad \frac{T(h)}{T(0)} = \left(\frac{P(h)}{P(0)}\right)^{\frac{R}{C_v+R}}$$

$$C_v = \frac{3}{2}R + \frac{2}{2}R$$

linear motions of molecules in 3 dimensions

2 rotational degrees of freedom treated classically

$$\boxed{\frac{T(h)}{T(0)} = \left(\frac{P(h)}{P(0)}\right)^{2/7}}$$

$$= e^{-\frac{2Mgh}{RT(0)}\frac{2}{7}}$$

$$T(h) = T(0)\, e^{-\frac{2Mgh}{k_B T_0}\frac{2}{7}}$$

Quals 2015

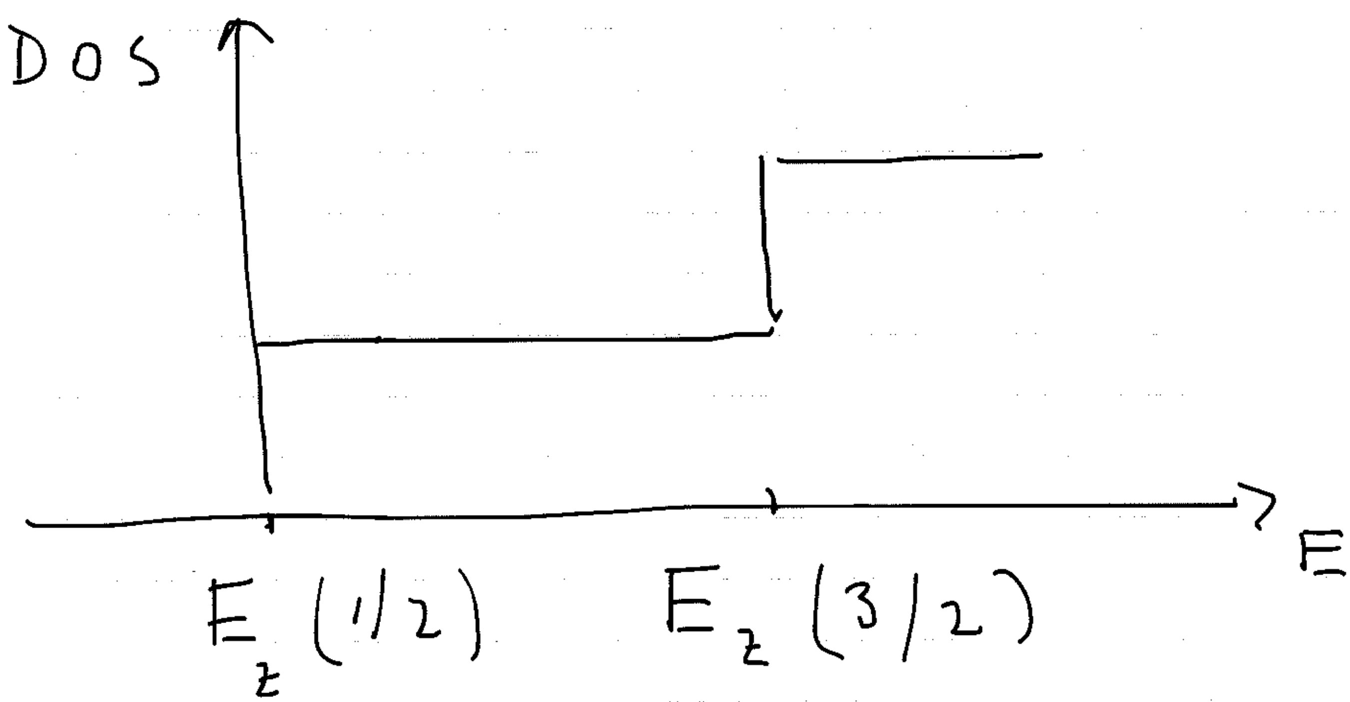
General - Condensed Matter

Solution

(a) For each value of j the DOS is

$$g(E) = \frac{m}{\pi \hbar^2}$$

The DOS is



(b) Calculate the Fermi energy

Only the $J = 0$ level is populated with

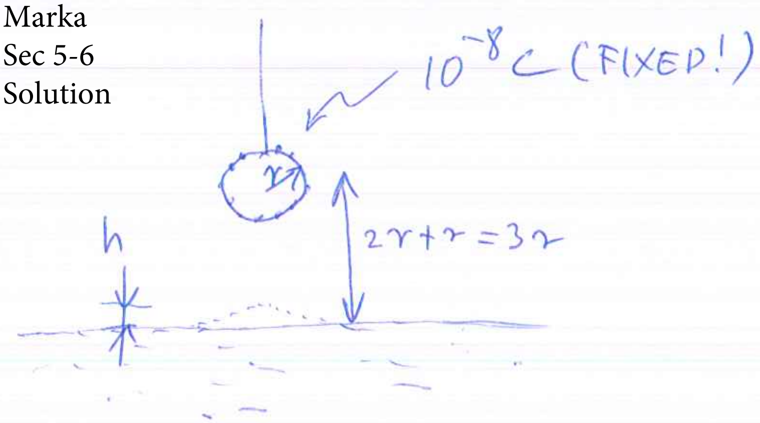
$$E_F = 0.00024\ eV = $$
$$= 0.24\ meV$$

As T is raised to T_2 the electron system becomes 'non-degenerate'.

(c) Levels with $J = 0, 1$ and 2 are populated. The Fermi energy is

$$E_F = 0.013\ eV = 13\ meV$$

Marka
Sec 5-6
Solution



$r = 1\,cm/2 = 0.005\,m$

$\rho_w = \rho = 1000\,kg/m^3$

$Q = 10^{-8}\,C$

$\boxed{h = ?}$

- SALT WATER IS ≈ A PLIABLE GOOD CONDUCTOR ⇒ CHARGES <u>MOVE</u>
- BALL IS INSULATOR ⇒ CHARGES ARE <u>FIXED</u>

$\Downarrow$

APPROXIMATE WITH <u>POINT</u> CHARGE
A <u>3r</u> HEIGHT ABOVE WATER

- WATER WILL RISE A BIT ONLY <u>$h \ll 3r$</u>! ⇒ SURFACE ≈ PLANE
- SHAPE OF WATER IS <u>NOT</u> THE QUESTION
- SURFACE TENSION IS <u>NEGLECTED</u> (I.E. BOND NUMBER IS LARGE ENOUGH, GRAVITY DOMINATES)
- FIELD UNDER THE BALL:

$$E = \frac{Q}{2\pi \varepsilon_0 (3r)^2}$$

$\Downarrow$

- CHARGE DENSITY UNDER THE BALL:

$$\sigma = \varepsilon_0 E = \frac{1}{2\pi} \frac{Q}{(3r)^2}$$

- BALANCE OF HYDROSTATICS & ELECTROSTATICS:

$$\sigma E_Q = \sigma \frac{Q}{4\pi \varepsilon_0 (3r)^2} = \frac{Q^2}{8\pi^2 \varepsilon_0 (3r)^4} = \rho_w g h$$

$\Downarrow$

$$\underline{\underline{h}} = \frac{Q^2}{8\pi^2 \varepsilon_0 (3r)^4 \rho_w g} = \frac{10^{-16}\,C^2}{79 * 8.85\times10^{-12}\frac{F}{m} * 5\times10^{-8}\,m^4 \times 10^3\,kg\,m^{-3}\,9.81\frac{m}{s^2}} =$$

$$= 2.9 * 10^{-4} \frac{C^2\,m\,s^2}{F\,m^4\,kg\,m^{-3}\,m} = 2.9*10^{-4} \frac{A^2 s^2 s^2 m}{\frac{s^4 A^2}{m^2 kg} m^2 kg} = \underline{\underline{0.29\,mm}}$$