Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 18, 2013
3:10PM to 5:10PM
General Physics (Part II)
Section 6.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 6 (General Physics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. The nucleus ^{57}Fe has an excited state at 14.4 keV that decays with a half-life time of 10^{-7} s under emission of a $\gamma-$quant.
   (a) What is the width of the γ-line?
   (b) What is the recoil energy of the free ^{57}Fe nucleus?
   (c) Why can the emitted γ not be absorbed by a second free ^{57}Fe nucleus at rest?
   (d) What minimum velocity does the second nucleus need to have in order to absorb the γ?
   (e) Assume the iron is bound in a macroscopic crystal lattice with an excitation frequency for lattive vibrations of 10^{-2} eV. How does this impact the absorption of the γ quant?
   (f) You build an experiment that aims to detect this gravitational redshift of photons with the Mössbauer Effect by moving a ^{57}Fe absorber crystal with respcet to a ^{57}Fe emitter crystal. What is the gravitational redshift $\frac{\Delta f}{f}$ of a photon that travels a distance d upwards in a gravitational potential?
   (g) Since your observed effect is small, how can you make sure that your observation is indeed due to gravitational redshift, that is, how can you eliminate some systematic uncertainties?

2. One peculiar property of the (electrically neutral) Higgs boson is that it has spin zero. While we do not fundamentally understand what spin is, we know it behaves like angular momentum. One of the important Higgs boson decay channels has it decaying to a pair of (charged) spin one W bosons. From an experimental point of view, this is a relatively background-free channel **if** both W bosons decay to a charged lepton and a neutrino, both of which have spin one-half. In this case, the dominant background is pair production of W bosons mediated by a (very) virtual spin one photon. A secondary background source is the production of a pair of charged leptons by a virtual spin one photon or a Z boson. W bosons also have a peculiar property: the W^- only decays to a "left-handed" charged lepton and a "right-handed" anti-neutrino, and the W^+ only to a "right-handed" charged lepton and a "left-handed" neutrino. Here left- and right-handed denote the alignment of the lepton spin with its momentum (either parallel or anti-parallel). How can conservation of angular momentum be used to distinguish pairs of W bosons produced in Higgs boson decays from those produced by virtual photons?

3. A balloon can be used to carry scientific instruments to high altitudes. If the volume of the balloon used is 30×10^6 feet^3, and the scientific instrument hanging below the balloon has a mass of 1000 kg, how high will the balloon go? Assume the gas in the balloon always has a density of $\rho_g = 10^{-5}$ kg/L, and the density of the atmosphere is $\rho(h) = \rho_0 \exp(-h/\alpha)$, where $\rho_0 = 10^{-3}$ kg/L and α, which is the scale height of the atmosphere, is equal to 7500 meters. You can also assume the mass of the balloon is negligible.

4. Let $f(\vec{x}, \vec{v}, t)$ be the phase space distribution of electrons in a highly ionized gas at temperature τ and in the presence of a weak constant (in space and time) external electric field $\vec{E} = E_0 \vec{e_x}$ with $\vec{e_x}$ a unit vector along the x-axis. The Boltzmann equation is

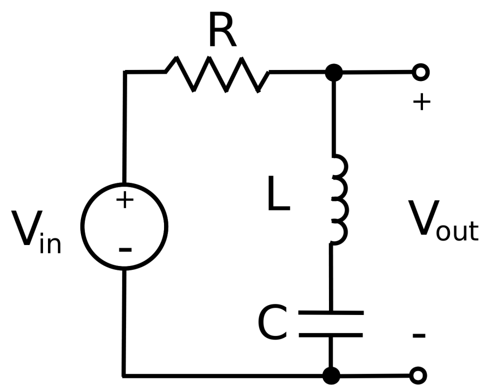
$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{x}} f + \vec{\alpha} \cdot \nabla_{\vec{v}} f = -\left(\frac{f - f_0}{t_c} \right) \tag{1}$$

where $\vec{\alpha} = -\frac{eE_0}{M} \vec{e}_x$ with $-e$ the charge of the electron, t_c is a constant (relaxation time) and

$$f_0 = m_0 \left(\frac{M}{2\pi\tau} \right)^{3/2} e^{-\frac{Mv^2}{2\tau}} \tag{2}$$

with M the electron mass and m_0 the density of electrons. If $\vec{J} = \sigma \vec{E}$ is the electromagnetic current, determine the conductivity, σ, in terms of e, M, m_0 and t_c.

5. Consider the electric circuit shown below. Suppose that an AC potential of the form $V_{\text{in}} = A \cos \omega t$ is applied at the input, as shown.



(a) Determine the amplitude of the output signal V_{out}.

(b) Determine the frequency for which the amplitude V_{out} will be a minimum.

(c) Considering the circuit's behavior, what might such a circuit be used for?

6. Two neutrons are bound into a dimer via gravitational attraction. Estimate the size and the binding energy of this "molecule". Based on their magnitudes, would you say these quantities can be experimentally measured? (The gravitational constant $G \sim 7 \times 10^{-11}$ $\mathrm{m}^3/(\mathrm{kg}{\cdot}\mathrm{s}^2)$.)

Mössbauer Effect

The nucleus ^{57}Fe has an excited state at 14.4 keV that decays with a half-live of 10^{-7} s under emission of a γ-quant.

1. What is the width of the γ-line?
2. What is the recoil energy of the free ^{57}Fe nucleus?
3. Why can the emitted γ not be absorbed by a second free ^{57}Fe nucleus at rest?
4. Which minimum velocity does the second nucleus need to have in order to absorb the γ?
5. Assume the iron is bound in a macroscopic crystal lattice with an excitation frequency for lattice vibrations of 10^{-2} eV. How does this impact the absorption of the γ quant?
6. You build an experiment that aims to detect this gravitational redshift of photons with the Mössbauer Effect by moving a ^{57}Fe absorber crystal with respect to a ^{57}Fe emitter crystal. What is the gravitational redshift $\frac{\Delta f}{f}$ of a photon that travels a distance d upwards in a gravitational potential?
7. Since your observed effect is small, how can you make sure that your observation is indeed due to gravitational redshift, that is, how can you eliminate some systematic uncertainties?

Solution

1. The width is

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.6 \times 10^{-16}\,\mathrm{eVs}}{1 \times 10^{-7}\,\mathrm{s}} = 6.6 \times 10^{-9}\,\mathrm{eV} \tag{1}$$

2. The recoil energy of the nucleus with mass $M \approx 54\,\mathrm{GeV/c^2}$ is

$$E_r = \frac{p_\gamma^2}{2M} = \frac{E_\gamma^2}{2Mc^2} = s\frac{(14\,\mathrm{keV})^2}{2 \times 54\,\mathrm{GeV}} = 1.9 \times 10^{-3}\,\mathrm{eV} \tag{2}$$

3. Absorbing the quant also leads to a recoil. Hence, the energy of the emitted γ-quant is $\sim 4\,\mathrm{meV}$ lower than the energy required to excite $^{57}\mathrm{Fe}$. The width of the γ-line is six orders of magnitude too small to compensate for the recoil energy.

4. The velocity of the absorbing nucleus in the direction of the emitting nucleus v has to be such that the frequencies $f = E/h$ satisfy

$$f_{\mathrm{absorb}} = \left(1 + \frac{v}{c}\right) f_{\mathrm{emit}} \tag{3}$$

$$\Rightarrow \quad \left(1 + \frac{v}{c}\right) = \frac{f_{\mathrm{absorb}}}{f_{\mathrm{emit}}} = \frac{E_\gamma + E_R}{E_\gamma - E_R} \tag{4}$$

$$\Rightarrow \quad \frac{v}{c} = 2.7 \times 10^{-7} = 80\,\mathrm{m/s} \tag{5}$$

5. The recoil energy is distributed over $\sim 10^{23}$ nuclei in the lattice, reducing the recoil energy below that of a phonon. Hence, no phonons are generated and the emission (or absorption) takes place without any recoil. The emitted γ quant can be absorbed by another iron nucleus.

6.

$$h\Delta f = mgd = \frac{hf}{c^2} gd \tag{6}$$

$$\Rightarrow \quad \frac{\Delta f}{f} = \frac{gd}{c^2} \approx 10^{-16} \times \text{ d in meters} \tag{7}$$

7. The distance between absorber and emitter can be chaged. Absorber and emitter can be swapped around.

1 HEP Problem

One peculiar property of the (electrically neutral) Higgs boson is that it has spin 0. While we do not fundamentally understand what spin is, we know it behaves like angular momentum. One of the important Higgs boson decay channels has it decaying to a pair of (charged) spin one W boson. From an experimental point of view, this is a relatively background-free channel **if** both W bosons decay to a charged lepton and a neutrino, both of which have spin one-half. In this case, the dominant background is pair production of W bosons mediated by a (very) virtual spin one photon. A secondary background source is the production of a pair of charged leptons by a virtual spin one photon of a Z boson. W bosons also have a peculiar property: the W^- only decays to a "left-handed" charged lepton and a "right-handed" antineutrino, and the W^+ only to a "right-handed" charged lepton and a "left-handed" neutrino. Here left- and right-handed denote the alignment of the lepton spin with its momentum (either parallel or anti-parallel). How can conservation of angular momentum be used to distinguish pairs of W bosons produced in Higgs boson decays from those produced by virtual photons?

1.1 Solution

In Higgs boson decays, the W boson spins have to compensate each other to add up to zero, so they are opposite. Now, if in a W^- decay the charged lepton is emitted preferentially along the W boson spin direction, if must be the reverse for a W^+ boson (because of left- vs right-handed decays). This means that for W bosons produced by Higgs boson decays, the charged leptons will be emitted preferentially in the same direction. This is not the case for the backgrounds.

Quals Problem #2

January 14-18, 2013

Problem

A balloon can be used to carry scientific instruments to high altitudes. If the volume of the balloon used is 30×10^6 feet^3, and the scientific instrument hanging below the balloon has a mass of 1000 kg, how high will the balloon go? Assume the gas in the balloon always has a density of $\rho_g = 10^{-5}$ kg/L, and the density of the atmosphere is $\rho(h) = \rho_o \exp(-h/\alpha)$, where $\rho_o = 10^{-3}$ kg/L and α, which is the scale height of the atmosphere, is equal to 7500 meters. You can also assume the mass of the balloon is negligible.

Suggested Solution to Quals Problem #2

January 14-18, 2013

Solution

At altitude, the sum of the forces equals zero, so

$$mg + \rho_g V g - \rho_o \exp(-h/\alpha) V g = 0. \tag{1}$$

Solving for h gives

$$h = \alpha \ \ln\left(\frac{\rho_o}{\rho_g + m/V}\right). \tag{2}$$

The volume of the balloon in liters is 8.5×10^8, so the altitude in meters is

$$h = 7500[\mathrm{m}] \ \ln\left(\frac{10^{-3}\,[\mathrm{kg/L}]}{10^{-5}\,[\mathrm{kg/L}] + 1000\,[\mathrm{kg}]/8.5 \times 10^8\,[\mathrm{L}]}\right) = 33,700 \ [\mathrm{m}] = 33.7 \ [\mathrm{km}]. \tag{3}$$

A. Mueller

Quals

General physics

Question: Let $f(\vec{x},\vec{v},t)$ be the phase space distribution of electrons in a highly ionized gas at temperature τ and in the presence of a weak constant (in space and time) external electric field $\vec{E} = E_0 \vec{e}_x$ with $\vec{e}_x$ a unit vector along the x-axis. The Boltzmann equation is

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla_{\vec{x}} f + \vec{\alpha}\cdot\nabla_{\vec{v}} f = -\left(\frac{f-f_0}{t_c}\right)$$

where $\vec{\alpha} = -\frac{eE_0}{M}\vec{e}_x$ with $-e$ the charge of the electron. t_c is a constant (relaxation time) and

$$f_0 = n_0\left(\frac{M}{2\pi\tau}\right)^{3/2} e^{-\frac{Mv^2}{2\tau}}$$

with M the electron mass and n_0 the density of electrons. If $\vec{J} = \sigma\vec{E}$ is the electromagnetic current determine the conductivity, σ, in terms of e, M, n_0 and t_c.

Answer: Since $\vec{E}$ does not depend on $\vec{x}$ or t neither will f. Then Boltzmann becomes

$$\vec{\alpha}\cdot\nabla_{\vec{v}} f = -\frac{f-f_0}{t_c} \text{ or } f \simeq f_0 - t_c\,\vec{\alpha}\cdot\nabla_{\vec{v}} f_0$$

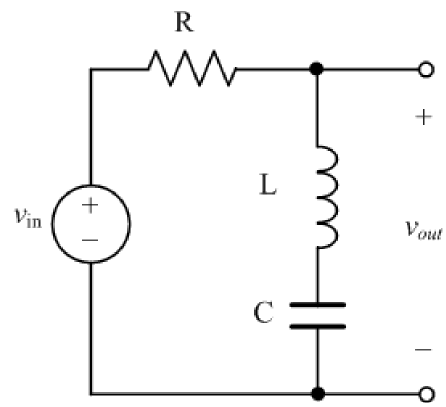
$$\vec{J} = \vec{e}_x J_x \text{ with } J_x = -e\int v_x f\, d^3x = e t_c \alpha \underbrace{\int v_x \nabla_{v_x} f_0\, d^3x}_{-\int f_0\, d^3x = -n_0}$$

$$J_x = -e\alpha t_c n_0 = \frac{e^2 t_c n_0 E_0}{M}$$

$$\boxed{\sigma = \frac{e^2 t_c n_0}{M}}$$

General Physics

Question: Consider the electric circuit shown below. Suppose that an AC potential of the form v_{in} = A cos ωt is applied at the input, as shown.



(a) Determine the amplitude of the output signal, v_{out} .
(b) Determine the frequency for which the amplitude of v_{out} will be a minimum.
(c) Considering the circuit's behavior, what might such a circuit be used for?

Soluion: (a) Consider the electric circuit as a complex voltage divider. Then

Vout/Vin = [jωL – j/ωC]/[R+ jωL – j/ωC]
= 1/[1 + jR/(1/ωC - ωL)]
= $1/[1 + j\omega RC/(1 - \omega^2 LC)]$

Therefore, in terms of amplitudes

Vout/Vin = modulus$(1/[1 + j\omega RC/(1 - \omega^2 LC)])$
= $1/\sqrt{[1+(\omega RC)^2/(1 - \omega^2 LC)^2]}$

(b) Minimum Vout/Vin = 0 for $(1 - \omega^2 LC) = 0 \rightarrow \omega = 1/\sqrt{(LC)} \rightarrow f = 1/[2\pi\sqrt{(LC)}]$

(c) Circuit can be used, for example, to eliminate a noise source at a particular frequency.

GENERAL PHYSICS – ORDER OF MAGNITUDE ESTIMATE

A neutron dimer.

Two neutrons are bound into a dimer via gravitational attraction. Estimate the size and the binding energy of this "molecule". Based on their magnitudes, would you say these quantities can be experimentally measured? (The gravitational constant $G \sim 7 \times 10^{-11}$ m^3/(kg$\cdot$s^2).)

GENERAL PHYSICS – ORDER OF MAGNITUDE ESTIMATE

A neutron dimer. SOLUTION.

The negative gravitational energy balances the positive quantum "particle-in-the-box" energy,

$$\frac{Gm_n^2}{L} \simeq \frac{h^2}{8m_n L^2}, \tag{1}$$

where L is the neutron separation. Hence,

$$L \simeq \frac{h^2}{8Gm_n^3} \sim 10^{23} \text{ m}. \tag{2}$$

L turns out to be 10^3 times larger than a typical galaxy!

The binding energy

$$E_b \simeq \frac{Gm_n^2}{L} \sim 10^{-87} \text{ J} \sim 10^{-54} \text{ Hz}, \tag{3}$$

which is tiny (one period of this frequency is $\sim 10^{54}$ s while the age of the universe is $\sim 10^{17}$ s!).