Columbia University
Department of Physics
QUALIFYING EXAMINATION
Friday, January 18, 2008
9:00 AM – 11:00 AM

General Physics (Part I)
Section 5.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing additional problems). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 5 (General Physics), Question 2; Section 5(General Physics) Question 7, etc.).

Do **NOT** write your name on your answer booklets. Instead clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on 8 ½ x 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good luck!!

1. Consider an ideal gas that consists of N particles. The gas has specific heat C_P, at constant pressure, and specific heat C_V, at constant volume, that are both independent of temperature.

(a) Find the work done in an isothermal change of volume from V_1 to V_2. In the isothermal process the gas is in contact with a thermal bath that is kept at a constant temperature T.

(b) Evaluate the quantity of heat Q absorbed in the process described in (a).

(c) The gas undergoes a change in volume from V_1 to V_2 that takes place at constant pressure (isobaric). The gas is thermally isolated and thus unable to exchange heat with its environment. Find the change in temperature $T_2 - T_1$.

2. In an Ar-ion laser, green light (515 nm) is emitted by a transition of Ar^+ ions in a discharge. The radiative lifetime of the relevant transition is τ_{sp} = 10 ns and its measured linewidth is $\Delta\nu$ = 3 GHz. The discharge has an effective temperature of T = 1000 K and is at a pressure of 0.1 atmosphere. The laser cavity consists of parallel plane mirrors separated by 1 m.

(a) Estimate the contribution to the linewidth from radiative decay (the natural linewidth).

(b) Estimate the contribution to the linewidth from Doppler broadening.

(c) Estimate the contribution to the linewidth from pressure broadening (collisions).

(d) Estimate how many different longitudinal modes the laser oscillates in.

(e) If the laser is operated in a pulsed mode, estimate the duration of the shortest pulse that it can support.

(f) If the average power of the laser is 10 W, what would be the peak power achievable for the laser operating in a pulsed mode in which a single pulse travels back and forth in the cavity? What would be the peak intensity (irradiance) of such a pulse focused to a diffraction-limited spot?

3. A particle slides without friction through a tunnel connecting two arbitrary points on the Earth's surface. The path of the tunnel is perfectly straight. You can model the Earth as a non-rotating sphere of uniform density.

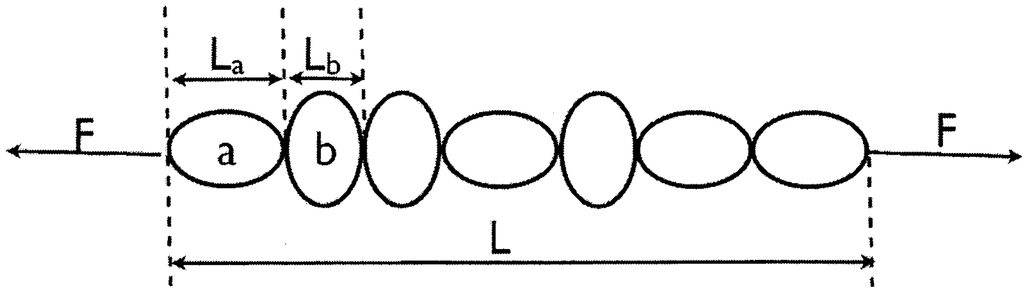
(a) Show that under the influence of gravity the particle undergoes simple harmonic motion in the tunnel. (You can assume the amplitude is small enough that the particle never leaves the tunnel.)

(b) What is the period of oscillation?

(c) Show that the period can be expressed in terms of Newton's constant and the density of the Earth, and is independent of where the endpoints of the tunnel are located.

4. Commencement ceremonies are displayed on huge LED-based television displays. Estimate how close you can get to these displays and still see an image and not the individual LEDs. Please feel free to select reasonable values for the necessary quantities.

5. Consider a one-dimensional chain consisting of N molecules which exist in two configurations, a and b, with corresponding energies E_a and E_b and lengths L_a and L_b. The molecules are in contact but do not otherwise interact (see figure). The chain is subject to a tensile force F and is in equilibrium with a thermal reservoir of temperature T.



(a) Write down the partition function for the system.

(b) Assume that $E_a > E_b$ and $L_a > L_b$. Make a sketch of the average length $< L >$ in the absence of tensile force, $F = 0$, as a function of temperature T. Give the high and low temperature limits and the characteristic temperature at which the changeover between the two limits occurs.

(c) Calculate the average length $< L >$ as a function of F and T.

6. Consider a spherical gas cloud in otherwise empty space. No forces act on the gas besides its own gravity and pressure. The gas molecules have mass m, and an unknown number density $n(r)$ that depends on the distance from the cloud's center (r). Assuming thermal equilibrium, that is, uniform temperature throughout:

(a) Derive the differential equation that $n(r)$ has to obey in order for a stationary configuration to exist.

(b) Solve the equation you derived assuming that $n(r)$ scales like a power of r.

(c) Compute the total mass of the cloud.

(d) What can you conclude about astrophysical gas clouds from your result in part (c)?

Pinczuk

Section 5 - General - I - Problem #1

(a) $$W = \int_{V_1}^{V_2} P\,dV = Nk_B T \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= Nk_B T \ln \frac{V_2}{V_1} = Nk_B T \ln \frac{P_2}{P_1}$$

because for an ideal gas

$$PV = Nk_B T$$

where $Nk_B = R$ = the ideal gas constant

(b) For an isothermal process there is no change in the internal energy:

$$\Delta U = W + Q = 0$$

$$Q = -W$$

(c)

$$\Delta U = C_V (T_2 - T_1)$$

$$\Delta U = W = P(V_2 - V_1)$$

$$T_2 - T_1 = \frac{P}{C_V}(V_2 - V_1)$$

AP.

Heinz - Problem 2

(a) $\Delta\nu_{sp} = \frac{1}{2\pi\tau_{sp}} = 16\,MHz$

(b) $\Delta\nu_D \approx 2\frac{\bar{v}_z}{c}\nu = \frac{2\bar{v}_z}{\lambda} \approx 2\frac{\sqrt{k_B T/M}}{\lambda} = \frac{2\times 680\,m/s}{511\,nm} = 2.7\,GHz$

(c) $\Delta\nu_{coll} = \frac{f_{coll}}{\pi}$

$f_{coll} \approx \bar{v}\pi d^2 n = \sqrt{3}\,\bar{v}_z \pi d^2 n$, with $d \approx 1\,Å$ atomic diameter

$\approx 110\,MHz$

$n = 6\times10^{23}/210\,\ell = 3\times10^{24}\,m^{-3}$

$\Delta\nu_{coll} \approx 35\,MHz$

(d) $\Delta\nu_{mode} = \frac{c}{2L} = 150\,MHz$

Number of accessible modes $\sim \frac{\Delta\nu}{\Delta\nu_{mode}} = 20$ (2X if you include polarization)

(e) By the time-frequency uncertainty relation

$\tau \geq (4\pi\Delta\nu)^{-1} = 26\,ps$

(f) The longest separation possible between the pulses is one cavity roundtrip of $2m/3\times10^8\,m/s = 6.7\,ns$. Then

$\bar{P} = P_{peak}\frac{25\,ps}{6.7\,ns} \Rightarrow P_{peak} = \bar{P}\frac{6.7\,ns}{25\,ps} = 2.7\,kW$

$I_{peak} = \frac{P_{peak}}{\lambda^2} = 1\times10^{16}\,W/m^2$

(Approximate answers are fine for all questions. Relations above are approximately correct for $\Delta\nu$ as FWHM)

Section 5, #3

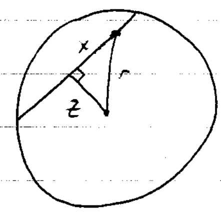
Kabat
JAN - 3 2008

General I #3 - falling through the earth

$$M(r) = \frac{4}{3}\pi r^3 \rho$$

$$F = \frac{G M(r) m}{r^2} = \frac{4}{3}\pi r \rho G m$$

$$V(r) = \int_0^r dr\, F = \frac{4}{6}\pi r^2 \rho G m = \frac{4}{6}\pi \rho G m (x^2+y^2+z^2)$$



energy conservation

$$\frac{1}{2}m\dot{x}^2 + \frac{4}{6}\pi \rho G m x^2 = \text{const.}$$

harmonic oscillator with $\omega^2 = \frac{4}{3}\pi \rho G$

period $\frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$ independant of tunnel endpoints

Sec 5, # 4
marka

Choose the following realistic ranges for the relevant parameters to define the range of the acceptable answers:

Aperture range for human eye (~pupil): $D = 2 - 8 \times 10^{-3}$ m

Wavelength range for visible light: $\lambda = 400 - 700 \times 10^{-9}$ m

Half pitch of LED displays (outdoor and indoor): $h = 3 - 8 \times 10^{-3}$ m

Minimum distance for diffraction limited seeing: L = ??? m

From the Rayleigh criterion and some trigonometry (small angles $\sin\alpha \approx \alpha$):

$$L \approx h D / 1.22 \lambda$$

So the lower bound can be as small as:

$$L_{MIN} \approx h_{MIN} D_{MIN} / 1.22 \lambda_{MAX} = 3\text{mm } 2\text{mm} / 1.22 \ 700 \text{ nm} = \mathbf{7\ m}$$

So the upper bound can be as large as:

$$L_{MAX} \approx h_{MAX} D_{MAX} / 1.22 \lambda_{MIN} = 8\text{mm } 8\text{mm} / 1.22 \ 400 \text{ nm} = \mathbf{131\ m}$$

However, the nominal distance in nice sunlight for decent outdoor displays for peak eye sensitivity:

$$L_{NOM} \approx h D / 1.22 \lambda = 8\text{mm } 3\text{mm} / 1.22 \ 550\text{nm} = \mathbf{35\ m}$$

Please note that often times not the diffraction limit is the limiting factor but the resolution of the retina.

Section 5, # 5

Solution: Millis Statistical mechanics

(a)

$$Z = \left(e^{-\frac{E_a - FL_a}{T}} + e^{-\frac{E_b - FL_b}{T}}\right)^N \tag{1}$$

(b) As $T \to 0$ all molecules will be in lower energy state, so $L = NL_b$. As $T \to \infty$ each molecule has equal probability of being in state a and state b so $L = 0.5N(L_a + L_b)$. Characteristic crossover scale is $T \approx E_a - E_b$.

(c)

$$\begin{aligned} < L > &= T\frac{\partial lnZ}{\partial F} \\ &= N\frac{\left(L_a e^{-\frac{E_a - FL_a}{T}} + L_b e^{-\frac{E_b - FL_b}{T}}\right)}{e^{-\frac{E_a - FL_a}{T}} + e^{-\frac{E_b - FL_b}{T}}} \end{aligned} \tag{2}$$

Sec 5, # 6
Nicolis

2008 Problems and Solutions – A. Nicolis

1. Consider a spherical gas cloud in otherwise empty space. No forces act on the gas besides its own gravity and pressure. The gas molecules have mass m, and an unknown number density $n(r)$ that depends on the distance from the cloud's center (r).

Assuming thermal equilibrium, that is uniform temperature throughout:

(a) Derive the differential equation that $n(r)$ has to obey in order for a stationary configuration to exist.

(b) Solve the equation you derived assuming that $n(r)$ scales like a power of r.

(c) Compute the total mass of the cloud.

(d) What does the last result mean?

Solution.

(a) The pressure is $P = n(r)kT$; the gravitational force acting on every molecule is

$$F_g(r) = G\frac{mM(r)}{r^2} = 4\pi G\frac{m^2}{r^2}\int_0^r n(r')r'^2 dr' \,, \tag{1}$$

directed inward. Balancing pressure gradient and gravitational pull we get

$$kT\frac{dn}{dr} = -4\pi G\frac{m^2}{r^2}n(r)\int_0^r n(r')r'^2 dr' \,. \tag{2}$$

We can get rid of the integral on the r.h.s. by deriving with respect to r (after an obvious manipulation).

$$kT\frac{d}{dr}\left(r^2\frac{1}{n(r)}\frac{dn}{dr}\right) = -4\pi G m^2\, n(r) r^2 \,. \tag{3}$$

(b) The ansatz $n(r) = A\, r^\alpha$ yields $n(r) \propto 1/r^2$.

(c)

$$M_{\rm tot} = m\, 4\pi\int_0^\infty n(r) r^2 dr = \infty \tag{4}$$

(d) That a realistic (=finite mass), self-gravitating gas cloud will not reach thermal equilibrium.