Columbia University
Department of Physics
QUALIFYING EXAMINATION
Friday, January 13, 2006
9:00 AM – 11:00 AM

General Physics (Part I)
Section 5.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing additional problems). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 5 (General Physics), Question 2; Section 5(General Physics) Question 7, etc.)

Do **NOT** write your name on your answer booklets. Instead clearly indicate your **Exam Letter Code**

You may refer to the single handwritten note sheet on 8 ½ x 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good luck!!

Problem 1 (Section 5 General)

An ideal gas of N atoms in a volume V is in thermal equilibrium with temperature T_i but is assumed to be isolated from other systems. At $t = 0$ all the atoms with kinetic energy larger than $\frac{1}{2}Mv^2 > \alpha\, k_B\, T_i > 0$ are allowed to escape the volume. After that the remaining atoms are assumed to come slowly to a new thermal equilibrium at temperature $T_f(\alpha)$ due to some unspecified weak interactions.

a) Find the dependence of $\mathrm{T}_f(\alpha)$ on α.

b) Find the asymptotic behavior of $T_f(\alpha)$ for very small $\alpha \ll 1$ and very large $\alpha \gg 1$.

Problem 2 (Section 5 General)

The specific heat, C_V, of a system is found to be independent of its volume, i.e., $(\partial C_V/\partial V)_T = 0$.

a) Write down the most general form of the Free Energy $F(T, V)$ that is compatible with this condition.

b) Write down the most general equation of state (i.e. P(V,T), with P the pressure) that is compatible with this condition.

Problem 3 (Section 5 General)

Compute the equilibrium ratio of the number of neutrons to protons in a neutron star. You can assume the electrons, neutrons and protons are all relativistic and degenerate inside the neutron star.

Problem 4 (Section 5 General)

Excitons in semiconductors are bound electron-hole pairs, typically generated after electrons and holes have been created by absorption of light and just before they recombine to emit again a photon. Assume that the conduction band and valence band of the semiconductor follow the dispersion relation

$$E = \pm\sqrt{\left((v\hbar k)^2 + \Delta^2\right)}$$

with v=c/200; c being the speed of light and Δ =1eV.

A) Determine the band edge electron mass, m_e, and the band edge hole mass, m_h.

B) Calculate the binding energy of an exciton in this material, assuming a dielectric constant of $\varepsilon = 16$ for the semiconductor.

C) At what temperatures would we observe excitons.

D) If you wanted to create *free* (non bound) electrons and holes in the semiconductor what is the minimal photon energy required in this example?

E) After formation of exciton and recombination of the electron with the hole, what is the resulting photon energy?

Problem 5 (Section 5 General)

An incompressible liquid is kept under pressure ($P_\circ$) by a movable piston. A gas bubble of radius $R_\circ$ is trapped within the liquid (see Figure 1).

(a.) What is the pressure (P) of the trapped gas?

(b.) Very roughly, what is the liquid's surface tension (σ) in terms of some of the other parameters given below?

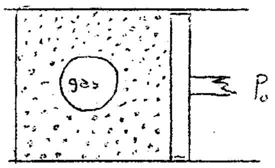


Figure 1: Incompressible liquid, gas bubble and piston.

Please use the following notation:

$R_\circ$ Bubble radius

L Heat of vaporization per unit mass of liquid

σ Surface tension of liquid

ρ_L Density of the liquid

ρ_g Density of the gas in part

m_g Mass of a gas molecule

m_L Mass of a liquid molecule

Problem 6 (Section 5 General)

The acceleration due to gravity on the surface of Mercury is $3.5\,\mathrm{m\,s^{-2}}$. The radius of Mercury is $2.4 \times 10^6\,\mathrm{m}$. Suppose that the atmosphere of Mercury were pure H_2 gas.

(a) What should the temperature be for the rms speed of the H_2 molecules to match the escape speed? Qualitatively, what is the effect on the temperature of the remaining gas in this situation?

(b) Would there be a similar effect if the actual temperature was less than the result in (a) ?

(c) If Mercury's atmosphere had two or more components, what would happen to the composition as a function of time ?

Statistical mechanics

IgorAleiner

NOV 2 2 2005 Sec 5

#1

Evaporating cooling

Ideal gas of atoms with established temperature T_i is kept isolated. At $t=0$, all the atoms with the kinetec energy larger than yT_i are removed from the system ($y>0$, is the numerical coefficient. After that the system is isolated ~~by~~ but weak interaction leads to the establishing new temperature at $t\to\infty$; $T(t\to\infty)=T_f$.

Find $T_f(y)$ for an arbitrary y (10pts).

Find the asymtotic behavior of $T_f(y)$ at $y\to 0$, and $y\to\infty$ (5pts)

Solution:

$$\mathcal{E}_i = \frac{3}{2}N_i T_i$$

$$\mathcal{E}_f = \frac{3}{2}N_f T_f;$$

$$\frac{N_f}{N_i} = \frac{\int_0^y dx\sqrt{x}\,e^{-x}}{\int_0^\infty dx\sqrt{x}\,e^{-x}};$$

$$\frac{\mathcal{E}_f}{\mathcal{E}_i} = \frac{\int_0^y dx\, x^{3/2}e^{-x}}{\int_0^\infty dx\, x^{3/2}e^{-x}};$$

$$\frac{T_f}{T_i} = \frac{\mathcal{E}_f}{\mathcal{E}_i}\frac{N_i}{N_f} = \frac{2}{5}\frac{\int_0^y dx\, x^{3/2}e^{-x}}{\int_0^y dx\, x^{1/2}e^{-x}} = \qquad (10pts)$$

$$= \begin{cases} 6/25\, y; & y\to 0; \\ 1-\frac{2}{?}y^{3/2}e^{-y}; & y\to\infty; \end{cases} \qquad (5pts).$$

Thermal physics

Igor Aleiner

NOV 2 2 2005 Sec 5 #2

The specific heat of some system does not depend on its volume, $\left(\frac{\partial C_V}{\partial V}\right)_T = 0$;

1) Write down the most general form of the Free energy $F(T,V)$ compatible with this condition. (8pts);

2) Write down the most general equation of state compatible with $\left(\frac{\partial C_V}{\partial V}\right)_T = 0$;

Solution:

1)
$$\left.\begin{aligned} C_V &= -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V \\ \left(\frac{\partial C_V}{\partial V}\right)_T &= 0; \end{aligned}\right\} \Rightarrow F = F_1(T) + F_2(V) + T F_3(V)$$

where F_1, F_2, F_3 are the arbitrary functions constrained by

$$\frac{\partial^2 F}{\partial V^2} > 0, \text{ i.e. } \boxed{\frac{\partial^2 F_2}{\partial V^2} + T\frac{\partial^2 F_3}{\partial V^2} > 0}$$

2); $P = -\left(\frac{\partial F}{\partial V}\right)_T$,

linear T dependence:

$$P = P_1(V) + T P_2(V);$$

$$\left(\frac{\partial P}{\partial V}\right)_T < 0$$

Subject: 2 questions for the Quals committee
From: Lam Hui <lhui@astro.columbia.edu>
Date: Wed, 23 Nov 2005 12:42:20 -0500 (EST)
To: lalla@phys.columbia.edu, lhui@phys.columbia.edu

To the Quals Committee,

Below please find two problems: one mechanics and one general.

Lam

Mechanics -

Problem:

Consider the motion of the earth around the sun. Let's approximate the orbit as circular. Suppose the sun very slowly loses its mass, from an original mass of M_1 to a mass of M_2. Suppose the initial radius of the orbit is R_1 and the eventual radius is R_2. What is R_2 in terms of the other parameters?

Solution:

The angular momentum is an adiabatic invariant. Therefore, $M_1 R_1 v_1 = M_2 R_2 v_2$, with $v_1 = \sqrt{G M_1/R_1}$ and $v_2 = \sqrt{G M_2/R_2}$. Hence, $R_2 = R_1 (M_1/M_2)^3$ i.e. the orbit expands under mass loss.

Sec 5 #3

General -

Problem:

Compute the equilibrium ratio of the number of neutrons to protons in a neutron star. You can assume the electrons, neutrons and protons are all relativistic and degenerate inside the neutron star.

Solution:
The relevant process is a neutron turning into a proton, an electron and an anti-neutrino, and vice versa.
Chemical equilibrium demands the equality of chemical potentials: $\mu_n = \mu_p + \mu_e$. The chemical potential of anti-neutrinos is zero because they can escape the neutron star.
The chemical potential of each specie is given simply by the Fermi energy, which equals the Fermi momentum in the relativistic regime, which is proportional to the density n to the one-third power.
Therefore, $n_n^{1/3} = n_p^{1/3} + n_e^{1/3}$.
Setting $n_p = n_e$ for charge neutrality then tells us $n_n/n_p = 8$.

Ses 5 # 4

Qualifier Question Physics 2005, Stormer, General Physics
11/23/05

Excitons in semiconductors are bound electron-hole pairs, typically generated after electrons and holes have been created by absorption of light and just before they recombine to emit again a photon.

Assume that the conduction band and valence band of the semiconductor follow the dispersion relation

$$E = \pm\sqrt{\left((v\hbar k)^2 + \Delta^2\right)}$$

with v=c/200; c being the speed of light and Δ =1eV.

A) Determine the band edge electron mass, m_e, and the band edge hole mass, m_h.

(3 points)

$m_{e,h} = \hbar^2 (\partial^2 E / \partial k^2)^{-1}$ **yields**

$m_{e,h} / m_0 = \Delta / v^2 = (200)^2 \Delta / m_0 c^2 \approx 40000x(1eV)/0.5MeV = 0.08$ **where m_o is free electron mass.**

B) Calculate the binding energy of an exciton in this material, assuming a dielectric constant of ε = 16 for the semiconductor.

(2 points)

Both masses are identical. Therefore like positronium: $E = \frac{m_0 e^4}{4\hbar^2} = 1/2Ry = 6.8eV$.

However, mass m_e=m_h=0.08 m_o and E-field is screened by ε. Hence E gets multiplied by $m_{e,h}/(m_o\varepsilon^2)$=0.08/256=3.1 x 10^{-4}. E_{ex}=2.1meV

C) At what temperatures would we observe excitons.

(1 point)

At a temperature smaller than ~2.1meV or ~25K. Otherwise the excitons cannot form; e and h are not bound.

D) If you wanted to create *free* (non bound) electrons and holes in the semiconductor what is the minimal photon energy requires in this example?

(1 point)

The bandgap energy of Δ=1eV.

stormer sect 5
\# 4

E) After formation of exciton and recombination of the electron with the hole, what is the resulting photon energy?

(1 point)

$E=\Delta-E_{ex}\sim 0.998eV$

F) The same semiconductor also contains impurities; donors as well as acceptors. Calculate the binding energy of carriers to these impurities and specify which carrier binds to which kind of impurity.

(2 points)

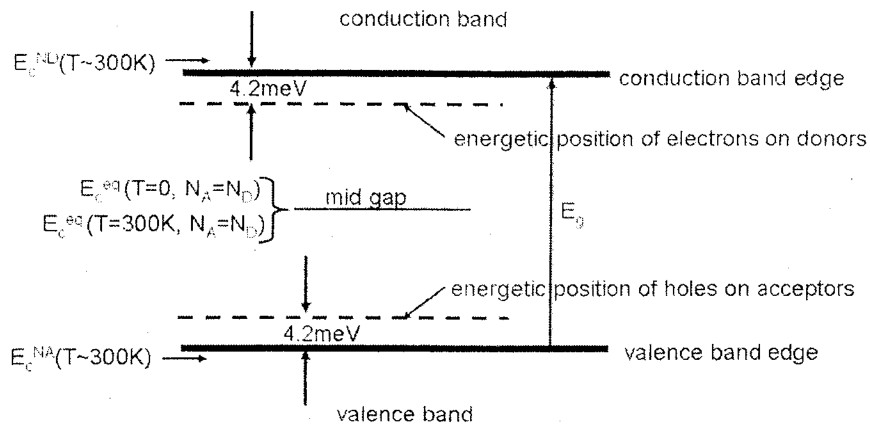
The equations are identical to the exciton case. However, now one of the charges is very heavy (acceptor, donor). Therefore the situation is equivalent to hydrogen and the binding energy is double as big as in the exciton case. $E_{D,A}$= 4.2meV. Electrons bind to donors. Holes bind to acceptors.

G) Make a graph of the energies versus some direction, x, in the semiconductor. Make this graph big, since the following questions require indicating several energetic positions within it. Start by indicating the position of the conduction band, the conduction band edge, the valence band, the valence band edge, the energy gap and its value, the energetic position of carriers bound to donors and carriers bound to acceptors, together with the energies calculated in B).

(2 points)

see graph

Stormer
sec 5 #4



H) Where would you locate the exciton in this graph? Explain in words.

(1 point)

The exciton cannot really be located in this graph, since it is a two-particle system. Sometimes it is indicated as a vertical arrow within the band gap of lengths $E=\Delta-E_{ex}$.

I) If the densities of donors, N_D, and acceptors, N_A, are the same, where would you locate the chemical potential, E_c^{eq} (T=0), at zero temperature and at E_c^{eq} (T~300K)?. Indicate both positions in the graph C).

(1 point)

At mid gap.

J) Also draw the approximate position of E_c^{ND}(T~300K) for N_D>>N_A and E_c^{NA}(T~300K) for N_D<<N_A.

(1 points)

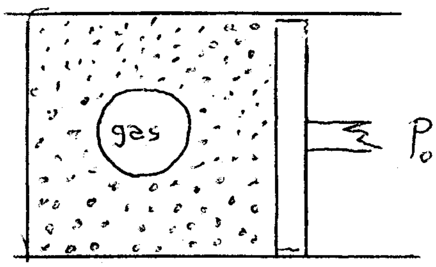
E_c^{ND}(T~300K) in conduction band, close to conduction band edge.
E_c^{NA}(T~300K) in valence band, close to valence band edge.

NOV 2 8 2005 — Sec 5 #5

M. Rudorman

General A

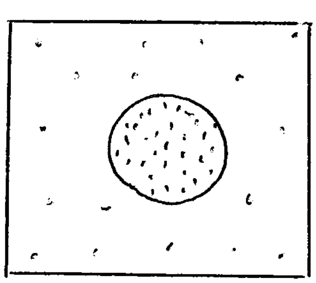
An (incompressible) liquid is kept under pressure (P_0) by a movable piston. Within this liquid a gas is trapped inside a bubble of radius R_0.



a) What is the pressure (P) of the trapped gas?

b) Very roughly, what is the liquid's surface tension (σ) in terms of some of the other parameters given below?

c) Consider a droplet of this same liquid ~~with~~ (radius R) in equilibrium with its own vapor. For $R \to \infty$, and the same ~~temp~~ temperature the equilibrium vapor pressure is P_1. What is the vapor pressure when R is finite? (Assume, if needed, $\sigma/R \ll P_1$)



$R_0 \equiv$ bubble radius

$L \equiv$ heat of vaporization per unit ~~volume~~ mass of liquid

$\sigma \equiv$ surface tension of the liquid

$\rho_L \equiv$ density of the liquid

$\rho_g \equiv$ density of the gas in a)

$m_g \equiv$ mass of gas molecule

$m_L \equiv$ mass of liquid molecule

$\rho_v \equiv$ density of vapor when $R \to \infty$ in c).

Greek rho

ρ_g

m_g

Rudeman General A

NOV 2 8 2005
Sec 5 #5

Answers

a) $\boxed{P = P_0 + \frac{2\sigma}{R}}$

$$-4\pi R^2 \underbrace{\Delta R}_{\Delta \text{Vol}} (P - P_0) + 8\pi R \underbrace{\Delta R}_{\Delta \text{Area}} \sigma = 0$$

or
very, very roughly, use dimensional analysis

$$P - P_0 \sim \frac{\sigma}{R}$$

b) Dimensional analysis using only the liquid's parameter

$$\boxed{\sigma \sim (L m_L)\left(\frac{\rho_L}{m_L}\right)^{2/3} \times f}$$

f = fraction of order unity

or

$$\sigma \sim \frac{\text{binding energy deficiency of a surface molecule}}{\text{surface area for each surface molecule}}$$

$$\sim \frac{f L m_L}{(m_L/\rho_L)^{2/3}}$$

$f < 1$,
typically $\sim \frac{1}{6}$
for cubic lattice

c) chemical potential equality is maintained so

$\mu_L = \mu_V \qquad \Delta\mu_L = \Delta\mu_L$

$$(v\Delta p - s\Delta T)_L = (v\Delta p - s\Delta T)_V$$

$\Delta T = 0$
$\Delta P_L \sim \frac{2\sigma}{R}$

$$\boxed{\therefore \Delta P_{vapor} \cong \frac{2\sigma}{R} \frac{\rho_{vap}}{\rho_{Liq}}}$$

Westerhoff
11/28/05
General

Problem 1

sec 5 #6

The acceleration due to gravity on the surface of Mercury is $3.5\,\mathrm{m\,s^{-2}}$. The radius of Mercury is $2.4 \times 10^6\,\mathrm{m}$. Suppose that the atmosphere of Mercury were pure H_2 gas.

(a) What would the temperature be so that the rms speed of the H_2 molecules matched the escape speed ? Qualitatively, what is the effect on the temperature of the remaining gas ?

(b) Would there be a similar effect if the actual temperature was less than the result in (a) ?

(c) If Mercury's atmosphere had two or more components, what would happen to the composition as a function of time ?

Problem 2 (10 points)

sec 4 #5

The detection of neutrinos from Supernova SN 1987A can be used to put an upper limit on the neutrino mass. Show that for two neutrino events with different energies E_1 and E_2, the arrival time difference on Earth is given by

$$\Delta t \simeq \left(\frac{Lm^2c^4}{2c}\right)\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right) ,$$

where L is the distance to the supernova, and m is the neutrino mass. Calculate an upper limit using typical values $E_1 = 10\,\mathrm{MeV}$, $E_2 = 20\,\mathrm{MeV}$ and the fact that the neutrino pulse from SN 1987A lasted less than 10 s and SN 1987A is 170 000 light years away. Can this limit compete with current limits from tritium beta decay ?

Sec 5 #6

Solutions

Problem 1:

(a) escape speed $\frac{1}{2} m v_{esc}^2 - G\frac{Mm}{R} = 0$

$$\Rightarrow \quad v_{esc} = \sqrt{2gR} \qquad g = 3.5\ \frac{m}{s^2}$$

average kinetic energy of H_2 is $\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$, so for $v_{rms} = v_{esc}$

$$\frac{1}{2} m\, 2gR = \frac{3}{2} kT \quad \Rightarrow \quad T = \frac{1}{3}\, \frac{2mgR}{k}$$

$$m = 2g/_{mol} \cdot \frac{mol}{6.02 \cdot 10^{23}}$$

So

$$T = \frac{1}{3}\, \frac{2 \cdot (3.5\, m s^{-2})(2g/mol)}{6.02 \cdot 10^{23}\, /_{mol}\ \ 1.38 \cdot 10^{-23}\, J/K}\ 2.4 \cdot 10^6 m$$

$$= \underline{\underline{1348\ K}}$$

As faster molecules escape, v_{rms} and T <u>decrease</u>.

(b) Yes, since the speed distribution has the speed of some H_2 molecules greater than v_{rms}, but the fraction is less, so H_2 molecules escape more slowly.

(c) The lighter component escapes more rapidly.

Columbia University
Department of Physics
QUALIFYING EXAMINATION
Friday, January 13, 2006
11:10 AM – 1:10 PM

General Physics (Part II)
Section 6.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing additional problems). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 6 (General Physics), Question 3; Section 6(General Physics) Question 6, etc.)

Do **NOT** write your name on your answer booklets. Instead clearly indicate your **Exam Letter Code**

You may refer to the single handwritten note sheet on 8 ½ x 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted. Questions should be directed to the proctor.

Good luck!!

Problem 1: Section 6 General Physics

Energetic photons are attenuated in "free space" due to collisions with cosmic microwave background (CMB) photons (and infrared photons that we will ignore) that result in e^+e^- pair production. In answering the following questions take the CMB temperature to be given by $k_B T = 2.5 \times 10^{-4}$ eV, the electron mass to be $m_e \approx 0.5$ MeV and the product $\hbar c = 200$ eV nm.

a) **[3 pts]** Estimate the minimum energy, E_{min}, at which a photon propagating through space will produce an e^+e^- pair assuming that the CMB photons have energy $k_B T$.

b) **[4 pts]** Find a symbolic expression for k_{min}, the minimum CMB photon momentum that can produce e^+e^- pairs when colliding with a propagating photon of energy E as a function of θ, the angle between the momentum vectors of the propagating photon and the CMB photon.

c) **[4 pts]** At photon energies above 10^{15} eV the photon-photon scattering is dominated by e^+e^- pair production at threshold. Suppose you are given the pair production cross-section at threshold, σ. Show symbolically how you would calculate the mean free path of a photon with enery E propagating through the universe assuming that it interacts with the full spectrum of CMB photons and only by pair production at threshold. You may leave your results in terms of an unevaluated integral.

d) **[4 pts]** Suppose $E = E_{\text{min}}/6$. Obtain an *order-of-magnitude estimate* for the mean free path of photons with this energy using $e^{-6} \approx 1/400$ and $\sigma \approx 1 \times 10^{-25}\text{cm}^2$. You may find it convenient to express your result in terms of parsecs, 1 parsec $\approx 3 \times 10^{16}$ m. **Beware, even when simplified, the integral in part c) cannot be completely evaluated analytically. You must find a way to approximate the integral.**

Problem 2: Section 6 General Physics

A capacitor with plate separation d is placed in an ideal gas of molecules at temperature T. The molecules have polarizability α. Find the ratio of gas pressures inside and outside the capacitor as a function of voltage V applied to it.

Problem 3: Section 6 General Physics

A total of N non-relativistic electrons are confined to a box of volume V. Suppose the electrons are in their ground state, meaning that they form a degenerate Fermi gas. Aside from the Pauli principle you can neglect interactions between electrons.

a) **[3 pts]** Compute the energy of the gas as a function of N and V.

b) **[3 pts]** Compute the pressure P exerted by the gas on the walls of the box, and evaluate the bulk modulus $B = -V\frac{\partial P}{\partial V}$.

Beside the electrons, suppose the box contains a lattice of atoms (which we have ignored up to this point). A longitudinal sound wave propagates through the box in the x direction. The wave can be characterized by a displacement field $\chi(x,t)$. This just means the atoms that are at position x in equilibrium (in the absence of a sound wave) have been displaced to position $x + \chi(x,t)$.

c) **[3 pts]** Show that the mass density in the box is $\rho = \rho_0/(1 + \frac{\partial \chi}{\partial x})$, where ρ_0 is the equilibrium mass density.

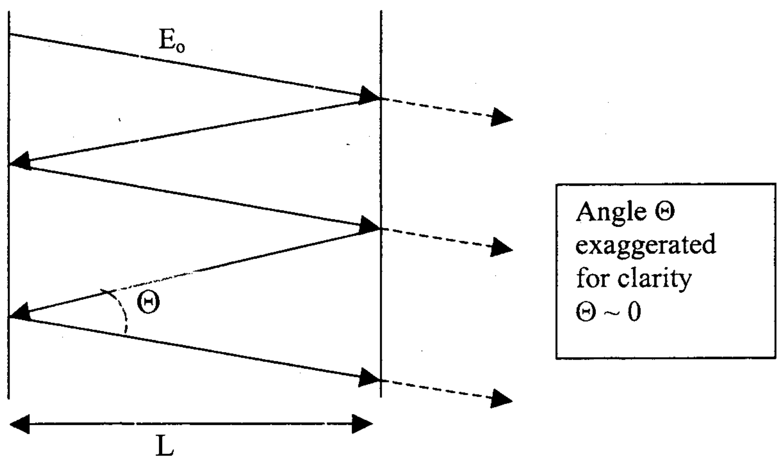
d) **[3 pts]** For small displacements show that $\chi(x,t)$ obeys the wave equation. Show that the speed of sound is $v_s = \sqrt{B/\rho_0}$.

d) **[3 pts]** The density of conduction electrons in copper is $n_e = 8.5 \times 10^{22}$ cm^3, and the mass density of copper is $\rho_0 = 9$ g/cm^3. Estimate the bulk modulus and speed of sound in copper.

Useful facts: $\hbar c \approx 2 \times 10^{-5}$ eV $\cdot$ cm, $m_e c^2 = 500$ keV, 1 eV/c$^2 \approx 2 \times 10^{-33}$ g.

Problem 4: Section 6 General Physics

Consider a cavity with mirrors of amplitude reflectivity r and reflection coefficient $r^2 = R \sim 0.99$ and a separation between the mirrors L. The laser beam continuously reflects between the mirror ends. The high reflectivity permits the beam in opposite directions to be considered of approximately equal amplitudes so that standing waves are generated which constitute the longitudinal modes of the cavity.



a) **[3 pts]** What is the phase change during one round trip of the laser cavity? Express your answer in terms of L and ν, the radiation frequency.
b) **[4 pts]** For what mode frequencies ν can standing waves be maintained in the cavity? Express your answer in terms of L. What is the separation Δν between the mode frequencies?
c) **[4 pts]** Find an expression for the intensity of radiation transmitted out the far end of the laser cavity in terms of L, ν and R.
d) **[4 pts]** As the frequency changes from the standing wave value, the intensity of radiation out of the end of the cavity decreases. Use your results from (a) and (c) to determine the shift in frequency over which the intensity drops to half its maximum value.

Problem 5: Section 6 General Physics

An interface between two materials A and B may be characterized by a surface *tension* σ_{AB} which is the energy cost per unit area of the interface.

Consider a spherical particle of mass density ρ_P and radius R placed in a cylindrical container of radius d filled with liquid to a height h. The mass density of the liquid is $\rho_L < \rho_P$.

Assume that the surface tension of the particle-liquid interface is σ_{PL}, that the surface tension of the liquid-air interface is σ_{LA} and that the surface tension of the particle-air interface vanishes: $\sigma_{PA} = 0$.

[15 pts] Find the height z of the particle above the bottom of the container. You may assume the container radius d is much greater than the particle radius R.

Problem 6: Section 6 General Physics

Consider a radioactive source which emits a positron in every decay. In ordinary matter, the positron is stopped within the source holder, and annihilates with an electron, which has a rest mass of 511 keV.

(a) [**5 pts**] What particle(s) will be emitted, and with what energy, in the annihilation process? How many particles are emitted? If there are multiple particles, is there any correlation in the directions of their emission? Let us define this particle(s) as particle **a**.

(b) [**5 pts**] In the case of a decay of ^{22}Na, the positron decay event leaves the system in an excited state of ^{22}Ne, which then decays with the emission of a gamma ray of 1.27 MeV. We denote this particle as particle **b**.

We want to distinguish between the particles **a** and **b**. What kind of particle detector shall we use for this purpose? Why can we distinguish between these particles?

(c) [**5 pts**] Suppose we have two detectors: detector A which detects particle **a**, selectively, and detector B which detects particle **b**, selectively. The time resolution of these detectors is Δt, which is much longer than the time interval of the successive decay events **a** and **b**. Suppose we have a system to measure the single rate of **a**-decay and the single rate of **b**-decay, by using the detectors A and B. We also can measure the rate of successive decays by taking the time coincidence of the A and B counter signals. Let us define the rates of these as R_a, R_b and R_{ab}. Show that we can determine the strength of the original radioactive source (i.e., N decay events per second) by using this information. Describe why and how we can do this.

Beloborodov
11/28/05
General

Sec 6 #2

Problem 2. A capacitor with plate separation d is placed in ideal gas of molecules with temperature T. The molecules have polarizability α. Find the ratio of gas pressures inside and outside the capacitor as a function of voltage V applied to it.

Solution: Electric field inside the capacitor is $E = V/a$. The dipole moment of a molecule inside the capacitor is $d = \alpha E$ and its potential energy in the electric field is $U = -Ed$. The ratio of gas densities inside and outside the capacitor is given by the Boltzmann factor,

$$\frac{n}{n_0} = \exp\left(-\frac{U}{kT}\right).$$

The temperature is everywhere the same and hence the ratio of gas pressures inside and outside the capacitor is

$$\frac{p}{p_0} = \frac{n}{n_0} = \exp\left(\frac{\alpha V^2}{kTa^2}\right).$$

Sec 6 #3

Dan Kabat
11/15/05

General

A total of N non-relativistic electrons are confined to a box of volume V. Suppose the electrons are in their ground state, meaning that they form a degenerate Fermi gas. Aside from the Pauli principle you can neglect interactions between electrons.

1. Compute the energy of the gas as a function of N and V.

2. Compute the pressure P exerted by the gas on the walls of the box, and evaluate the bulk modulus $B = -V\frac{\partial P}{\partial V}$.

Beside the electrons, suppose the box contains a lattice of atoms (which we've ignored up to this point). A longitudinal sound wave propagates through the box in the x direction. The wave can be characterized by a displacement field $\chi(x,t)$. This just means the atoms that are at position x in equilibrium (in the absence of a sound wave) have been displaced to position $x + \chi(x,t)$.

3. Show that the mass density in the box is $\rho = \rho_0/(1 + \frac{\partial \chi}{\partial x})$ where ρ_0 is the equilibrium mass density.

4. For small displacements show that $\chi(t,x)$ obeys the wave equation. Show that the speed of sound is $v_s = \sqrt{B/\rho_0}$.

5. The density of conduction electrons in copper is $n_e = 8.5 \times 10^{22}/\text{cm}^3$, and the mass density of copper is $\rho_0 = 9\,\text{g/cm}^3$. Estimate the bulk modulus and speed of sound in copper.

Useful facts: $\hbar c \approx 2 \times 10^{-5}\,\text{eV}\cdot\text{cm}$, $m_e c^2 \approx 500\,\text{keV}$, $1\,\text{eV}/c^2 \approx 2 \times 10^{-33}\,\text{g}$.

Sec 6 #3

General problem solution

Box of volume L^3, $\psi_{\vec{n}} = e^{i 2\pi \vec{n}\cdot\vec{x}/L}$

$$E_{\vec{n}} = \frac{\hbar^2}{2m}\frac{4\pi^2|\vec{n}|^2}{L^2}$$

$$E = 2\int d^3n\, E_{\vec{n}} \qquad (\times 2 \text{ for two spins})$$

$$= 2\frac{\hbar^2}{2m}\frac{4\pi^2}{L^2}\int_0^{n_{max}} 4\pi n^2 dn \cdot n^2$$

$$= 2\frac{\hbar^2}{2m}\frac{4\pi^2}{L^2}\frac{4\pi}{5} n_{max}^5$$

$$N = 2\int_0^{n_{max}} d^3n = 2\cdot\frac{4}{3}\pi n_{max}^3$$

$$\Rightarrow E = \frac{16\pi^3}{5}\frac{\hbar^2}{mL^2}\left(\frac{3N}{8\pi}\right)^{5/3} = \frac{16\pi^3}{5}\frac{\hbar^2}{m}\frac{1}{V^{2/3}}\left(\frac{3N}{8\pi}\right)^{5/3}$$

$$P = -\frac{\partial E}{\partial V} = \frac{32\pi^3}{15}\frac{\hbar^2}{m}\frac{1}{V^{5/3}}\left(\frac{3N}{8\pi}\right)^{5/3}$$

$$B = -V\frac{\partial P}{\partial V} = \frac{5}{3}P$$

density?

$$dm = \rho_0 L^2 dx = \rho L^2\left(dx + \frac{\partial \chi}{\partial x}dx\right) \qquad \text{mass conservation}$$

$$\Rightarrow \rho = \frac{\rho_0}{1+\frac{\partial\chi}{\partial x}}$$

wave equation?

$$\text{Newton: } dF = \rho L^2 dx\, \ddot{\chi} = -\frac{\partial P}{\partial x} dx\, L^2$$

$$\rho\ddot{\chi} = -\frac{\partial P}{\partial x} = -\frac{\partial P}{\partial \rho}\frac{\partial\rho}{\partial x} = \frac{V}{\rho}\frac{\partial P}{\partial V}\frac{\partial \rho}{\partial x} \qquad \text{since } \frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$\rho\ddot{\chi} = -\frac{B}{\rho}\frac{\partial\rho}{\partial x}$$

$$\frac{\rho_0}{1+\frac{\partial\chi}{\partial x}}\ddot{\chi} = -\frac{B}{\rho_0}\left(1+\frac{\partial\chi}{\partial x}\right)\frac{\partial}{\partial x}\frac{\rho_0}{1+\frac{\partial\chi}{\partial x}}$$

$$\text{linearize} \Rightarrow \ddot{\chi} = +\frac{B}{\rho_0}\frac{\partial^2\chi}{\partial x^2}$$

sound speed $v_s = \sqrt{B/\rho_0}$

Sec 6 #3

For copper $B = \frac{32\pi^3}{15} \frac{\hbar^2 c^2}{m_e c^2} \left(\frac{3 n_e}{8\pi}\right)^{5/3}$

$\approx 2.5 \times 10^{23}\ eV/cm^3$

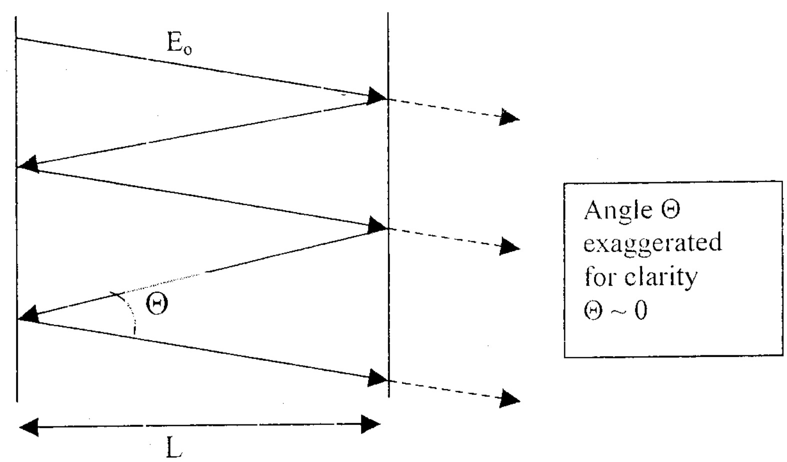
$v_s = \sqrt{\frac{B}{\rho_0}} = \left(\frac{2.5 \times 10^{23}\ eV}{9g}\right)^{1/2} \approx 2100\ m/s$

Sec 6
#4

Chuck Hailey's 2006 Quals problem (typed by Elena)
12/5/05

General Problem:

Consider a laser cavity with mirrors of amplitude reflectivity r and reflection coefficient $r^2 = R \sim 0.99$ and a length between the mirrors L. The laser beam continuously reflects between the mirror ends. The high reflectivity permits the beam in opposite directions to be considered of approximately equal amplitudes so that standing waves are generated which constitute the longitudinal modes of the cavity.



a) What is the phase change during one round trip of the laser cavity? Express your answer in terms of L and ν, the radiation frequency.
b) For what mode frequencies ν can standing waves be maintained in the cavity? Express your answer in terms of L. What is the separation $\Delta\nu$ between the mode frequencies?
c) Find an expression for the intensity of radiation transmitted out the far end of the laser cavity in terms of L, ν and R.
d) As the frequency changes from the standing wave value, the intensity of radiation out of the end of the cavity decreases. Use your results from (a) and (c) to determine the shift in frequency over which the intensity drops to half its maximum value.

QM Problem solution:

Hailey
Sec 6
4

General problem solution:

a.) $\phi = -2KL + 2\alpha$ $\quad \alpha$ = phase change at mirror

$\phi = -\frac{4\pi L\nu}{c} + 2\alpha$

b.) $2\pi m = \frac{4\pi L \nu}{c} - 2\alpha$

$\nu_m = \frac{mc}{2L} + \frac{\alpha c}{2\pi L}$

c.) $E_t = E_o t(1 + r^2 e^{-i\phi} + r^4 e^{-2i\phi} + \dots)$ $\quad t$ = Amplitude transmission

$E_t = \frac{E_o t}{1 - Re^{-i\phi}}$ $\quad R \equiv r^2$

$\left|E_t / E_o t\right|^2 = \frac{1}{(1-Re^{-i\phi})(1-Re^{i\phi})}$

$(1-Re^{-i\phi})(1-Re^{i\phi}) = 1 + R^2 - 2R\cos\phi = 1 + R^2 - 2R(1 - 2\sin^2\frac{\phi}{2})$

$= (1-R)^2 + 4R\sin^2\phi/2$

Absorbing $(1-R)^{-2}$ into I_{max}

$I_t = \frac{I_{max}}{1 + \frac{4R\sin^2\phi/2}{(1-R)^2}}$ $\quad \phi$ = from part (a)

d.) $I_t \to \frac{I_{max}}{2}$ when $\frac{4R\sin^2\delta\phi/2}{(1-R)^2} = 1$

where $\delta\phi$ is shift from peak value $2\pi m$

since $\frac{4R}{(1-R)^2} >> 1$ $\quad \delta\phi << 1$ $\quad 4R\sin^2\frac{\delta\phi}{2} \simeq \frac{4R}{4}\delta\phi^2$

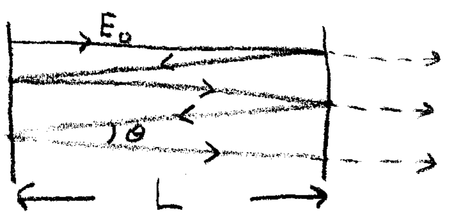
$\delta\phi = \frac{1-R}{\sqrt{R}} = \frac{4\pi L}{c}\delta\nu$ $\quad \delta\nu = \frac{c}{4\pi L}\frac{1-R}{\sqrt{R}}$ Ans
2

Sec 6 #4

NOV 2 2 2005

General: Ariley

Consider a laser cavity with mirrors of amplitude reflectivity r and reflection coefficient $r^2 = R \sim 0.99$ and a length between the mirrors L. The laser beam continuously reflects between the mirror ends. The high reflectivity permits the beams in opposing directions to be considered of approximately equal amplitude so that standing waves are generated which constitute the longitudinal modes of the cavity.



Angle θ exaggerated for clarity; $\theta \approx 0$

a) What is the phase change during one round trip of the laser cavity? Express your answer in terms of L and v, the radiation frequency.
b) For what mode frequencies v can standing waves be maintained in the cavity? Express your answer in terms of L. What is the separation Δv between the mode frequencies?
c) Find an expression for the intensity of radiation transmitted out the far end of the laser cavity in terms of L, v and R.
d) As the frequency changes from the standing wave value, the intensity of radiation out the end of the cavity decreases. Use your results from (a) and (c) to determine the shift in frequency over which the intensity drops to half its maximum value.

Solution general: Hailey

NOV 22 2005
Sec 6 #4

a.) $\phi = -2KL + 2\alpha$ $\quad \alpha$ = phase change at mirror

$$\phi = -\frac{4\pi L \nu}{c} + 2\alpha$$

b.) $2\pi m = \frac{4\pi L \nu}{c} - 2\alpha$

$$\nu_m = \frac{mc}{2L} + \frac{\alpha c}{2\pi L}$$

c.) $E_t = E_0 t (1 + r^2 e^{-i\phi} + r^4 e^{-2i\phi} + \ldots)$ $\quad t$ = Amplitude transmission

$$E_t = \frac{E_0 t}{1 - Re^{-i\phi}} \qquad R \equiv r^2$$

$$\left|\frac{E_t}{E_0 t}\right|^2 = \frac{1}{(1-Re^{-i\phi})(1-Re^{i\phi})}$$

$$(1-Re^{-i\phi})(1-Re^{i\phi}) = 1 + R^2 - 2R\cos\phi = 1 + R^2 - 2R(1 - 2\sin^2\tfrac{\phi}{2})$$
$$= (1-R)^2 + 4R\sin^2\phi/2$$

Absorbing $(1-R)^{-2}$ into I_{max}

$$I_t = \frac{I_{max}}{1 + \frac{4R\sin^2\phi/2}{(1-R)^2}} \qquad \phi = \text{from part (a)}$$

d.) $I_t \to \frac{I_{max}}{2}$ when $\frac{4R\sin^2\delta\phi/2}{(1-R)^2} = 1$

where $\delta\phi$ is shift from peak value $2\pi m$

since $\frac{4R}{(1-R)^2} \gg 1$ $\quad \delta\phi \ll 1$ $\quad 4R\sin^2\frac{\delta\phi}{2} \approx \frac{4R}{4}\delta\phi^2$

$$\delta\phi = \frac{1-R}{\sqrt{R}} = \frac{4\pi L}{c}\delta\nu \qquad \delta\nu = \frac{c}{4\pi L}\frac{1-R}{\sqrt{R}} \quad \frac{\text{rms}}{2}$$

NOV 2 1 2005

Sec 6 #5

Millis General 06 Quals Problem

An interface between two materials A, B may be characterized by a *surface tension* σ_{AB} which is the energy cost per unit area of the interface.

Consider a spherical particle of mass density ρ_P and radius R placed in a cylindrical container of radius d filled with liquid to a height h. The mass density of the liquid is $\rho_L < \rho_P$.

Assume that the surface tension of the particle-liquid interface is σ_{PL}, of the liquid-air interface is σ_{LA} while the surface tension of the particle-air interface vanishes: $\sigma_{PA} = 0$.

Please find the height z of the particle above the bottom of the container.

You may assume the container radius d is much greater than the particle radius R.

NOV 2 1 2006

SEC 6
5

Millis General 06 Quals Problem with Solution

An interface between two materials A, B may be characterized by a *surface tension* σ_{AB} which is the energy cost per unit area of the interface.

Consider a spherical particle of radius R placed in a cylindrical container of radius d filled with liquid to a height h.

Assume that the surface tension of the particle-liquid interface is σ_{PL}, of the liquid-air interface is σ_{LA} while the surface tension of the particle-air interface vanishes: $\sigma_{PA} = 0$.

Please find the position of the center of mass of the particle with respect to the surface of the liquid. You may assume the container radius d is much greater than the particle radius R, neglect gravity and assume that the surface tension of the liquid-air interface is positive.

Solution

There are three cases: particle on surface, particle at bottom of container, particle partly submerged. Choose zero of energy to be state in which particle is on top of liquid. Large d limit means may neglect energy cost of displaced liquid.

- Particle on top of liquid: energy 0.
- Particle submerged: energy of interface $E = 4\pi R^2 \sigma_{PL}$.
- Particle partly submerged. Center of mass moved down a distance L. Area submerged is $2\pi LR$. Liquid-air interface lost is $\pi L(2R - L)$. Total energy is

$$E = 2\pi LR\sigma_{PL} - \sigma_{LA}\pi L(2R - L)$$

This energy is minimized at $L^* = \left(1 - \frac{\sigma_{PL}}{\sigma_{LA}}\right) R$; minimum energy is $E^* = -\frac{1}{4}\sigma_{LA}L^{*2} = -\frac{1}{4}\frac{(\sigma_{LA}-\sigma_{PL})^2}{\sigma_{LA}}R^2$.

Therefore if $\sigma_{PL} > \sigma_{LA}$ the particle is expelled from the liquid while if $\sigma_{PL} < 0$ the particle is fully submerged. For intermediate values, the particle is partly immersed.

Tomo Uemura

General II
Problem 6. Section 6. JAN 17 2006

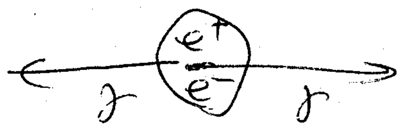
solution

a)

Positron Annihilation

$e^- + e^+ \rightarrow 2\gamma$ 511 keV each

Opposite direction



b) Both are γ-rays 511 keV
1.27 MeV

To distinguish energy,
use NaI scintillator + Photo Multiplyer,
or Ge detector

Energy deposition proportional to
pulse height

c) Originally N decays per sec

$R_a = 2 \cdot N \cdot (\text{detection efficiency } \varepsilon_a) \cdot (\text{solid angle } \Omega_a / 4\pi)$

$R_b = N \cdot \varepsilon_b \cdot \Omega_b / 4\pi$

$R_{ab} = N \cdot \varepsilon_b \cdot \Omega_b / 4\pi \cdot 2 \cdot \varepsilon_a \cdot \Omega_a / 4\pi$

$\therefore \quad N = R_a \cdot R_b \,/\, R_{ab}$