I will discuss the theory behind, and constraints on, interactions between the dark sectors of cosmology. In the first part of the talk, I will discuss the relationship between fluid models and the field theoretical models that underlie such descriptions. This question is particularly important in light of suggestions that such interactions may help alleviate a number of current tensions between different cosmological datasets. I will describe how to construct consistent field theory models for an interacting dark sector that behave exactly like the coupled fluid ones, even at the level of linear perturbations, and can be trusted deep in the nonlinear regime.

In the second part of the talk, I will focus on robust constraints on such models that can be obtained using only mildly nonlinear scales. I will show that lensing and clustering of galaxies in combination with the Cosmic Microwave Background (CMB) is capable of probing the dark sector coupling to the few percent level for a given class of models, using only linear and quasi-linear Fourier modes. These scales can, in principle, be described by semi-analytical techniques such as the effective field theory of large-scale structure.