This talk is devoted to Berry phases that appear in unitary representations of asymptotic symmetry groups in general relativity. These phases arise when a coherent state is acted upon by symmetry transformations that trace a closed path in the group manifold, and they can be evaluated exactly even when the group is infinite-dimensional. We apply these ideas to the Virasoro and BMS groups; seeing their representations as particles dressed with boundary gravitons, the associated Berry phases generalize Thomas precession and provide, in principle, observable signatures of asymptotic symmetries.